MAGNETIC TAPE RECORDER SPECTRAL PURITY

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Summary The data recovery and reduction processes of various telemetering systems have clearly demonstrated the critical role of flutter and time errors in instrumentation recording and reproduction. One of the effects of these errors is that of "frequency smearing" of individual recorded/reproduced sine wave components. The advent of precision tape speed servo control has reduced these errors to a point where a detailed examination of the residual effect becomes both possible and profitable. The detailed measurements of the resultant spectrum of a recorded/reproduced sine wave component are described. These are quantitatively analyzed and interpreted in terms of the flutter and time error characteristics of the recording/reproduction system.

Introduction and General Definitions A recent exercise in our Laboratory was the measurement and interpretation of the resultant spectrum of a sinusoid recorded and played back on a tape recorder/reproducer (pair) having a known combined record/playback flutter/time base error characteristic. In examining the literature in this area, it was found that much has been written around this subject. However, the relationships required to interpret the results were not displayed in an explicit form in terms of the flutter and time base error performance characteristics of the recorder/reproducer familiar to the instrumentation tape recorder user.

Flutter is defined as the variation of tape speed in the combined recording/reproducing process. This speed variation results in an instantaneous frequency modulation of a signal sine wave component. This frequency modulation has the effect of removing energy from a fundamental frequency and redistributing it into flutter noise sidebands. Even though conventional frequency response tests show uniform high frequency response, narrow band analysis reveals an attenuation of the higher recorded frequencies.

The time base error (TBE) is defined as the total time error caused by speed variations, measured from a reference point. It is due to the integrated effect of the flutter, and is related to the flutter-produced phase modulation of a signal sine wave component.

Abramson[1] has stated the general features of the resulting spectrum: "For small values of the mean-square modulating signal, it is known 2 that the spectrum -- approaches that of a cosine amplitude - modulated by the same signal, --. For large (but definite) values, the spectrum -has a Gaussian shape centered about the carrier frequency."

It is the purpose of this paper to re-state the analyses behind these statements in a form convenient to the instrumentation tape recorder user, and to work through a pair of specific examples to illustrate the use of the results. The results are presented for two limiting performance regions, and are displayed without formal proof. A partial list of references are given; the reader is invited to verify the stated relationships by referring to these papers and re-casting their results in terms of the parameters of interest to him.

Detailed Definition of Parameters In order to establish quantitative results we start with the defining equations and experimental techniques for flutter and time base error (TBE).

Flutter The output frequency of the combined record/playback process is given by

$$\text{Output Frequency} = \omega_o \,[1 + g_{RP}(t_p)] \qquad (1)$$

where $g_{RP}(t_p)$ = fractional* combined record/playback flutter evaluated at time of playback, t_p and $\omega_o = 2\pi f_o$ = angular frequency of recorded sinusoid.

The power spectral density of $g_{RP}(t_p)$ may be measured by the use of a discriminator operating on the carrier frequency modulated by the flutter. The power spectral density (P.S.D.) of $g_{RP}(t_p)$ is defined as the power in the normalized output of the discriminator observed in a one cycle slot at the frequency f.

Thus

$$\text{PSD of } g_{RP}(t_p) = \Phi_G(f) \qquad (2)$$

The mean square flutter is defined as the total power in the discriminator output

i.e.,

$$\sigma_F^2 = \int_0^\infty \Phi_G(f)df \qquad (3)$$

Time Base Error Consider the situation where we record two events separated in time by T seconds. Due to the flutter of the tape recorder, the two events are separated by a time T + δ (rather than T) when the tape is played back. The difference, δ, between the record and playback time-intervals is defined as the time base error difference (TBED).

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* Fractional flutter is the familiar percent flutter expressed in decimal form rather than in percent form.

In general, δ is a function of the original time interval, T, as well as being a function of the tape recorder flutter characteristics during recording and playback.

When the flutter spectrum contains no DC or very low frequency components, the mean square value of δ approaches a finite limit as the time interval T becomes large. This limit is defined as the mean square TBE.

$$\Delta^2 = \text{mean square TBE} = \lim_{T \to \infty} \int_0^\infty \frac{\sin^2 \pi f T}{(\pi f)^2} \Phi_G(f)\,df \tag{4}$$

Equation (4) cannot be evaluated explicitly for a generalized $\Phi_G(f)$. However, (4) may be rewritten as

$$\Delta^2 = \lim_{T \to \infty} 2\int_0^\infty (1 - \cos 2\pi f T) \frac{\Phi_G(f)}{(2\pi f)^2}\,df \tag{5}$$

As long as $\Phi_G(f)$ falls off at low frequencies, at least as fast as f^2, $\frac{\Phi_G(f)}{(2\pi f)^2}$ has a finite upper bound. Then, under the assumption of very large T, the cosine term in the integrand will go through many cycles of oscillation for only a very small percentage change in the f^2 term in the integrand. Consequently, the rapid cycling of the cosine term will average out to zero, and the contribution of the cosine term to the integral may be considered insignificant.

For example, if

$$\Phi_G(f) = \Phi_1 \frac{f^2/f_1^2}{1 + f^2/f_1^2} \quad \text{i.e., simple RC high-pass,} \tag{6}$$

where Φ_1 is a constant level,

$$\begin{aligned}\Delta^2 &= \lim_{T \to \infty} \frac{1}{\pi^2 f_1^2} \int_0^\infty \frac{\sin^2 \pi f T}{1 + f^2/f_1^2} \Phi_1\,df \\ &= \lim_{T \to \infty} \frac{\Phi_1}{4\pi f_1} \left[1 - e^{-2\pi f_1 T}\right] = \frac{\Phi_1}{4\pi f_1} \; .\end{aligned} \tag{7}$$

Thus we can state that as long as $\Phi_G(f)$ contains no DC or very low frequency components,

$$\Delta^2 = 2\int_0^\infty \frac{\Phi_G(f)}{(2\pi f)^2}\,df \; . \tag{8}$$

$\frac{\Phi_G(f)}{(2\pi f)^2}$ is seen to be the spectrum of the integrated flutter.

If we let

$$G(f) = \frac{\Phi_G(f)}{(2\pi f)^2} , \quad (9)$$

then

$$\Delta^2 = 2\int_0^\infty G(f)\, df \quad (10)$$

Thus we have at our disposal

1. Δ^2 = mean square TBE.
2. σ_F^2 = mean square fractional flutter.
3. $\Phi_G(f)$ = power spectrum of the fractional flutter.
4. $G(f)$ = power spectrum of the integrated fractional flutter.

These performance characteristics of the recorder/reproducer are characteristic of a given recorder at a given tape speed.

Analytical Results and Examples When a sinusoid of frequency f_o is recorded and reproduced, the resultant output power spectrum depends upon f_o and these four factors. The character of this spectrum is well defined in two asymptotic cases and is a mixture in the transition region. These two cases are for $2\pi\Delta f_o \ll 1$ and for $2\pi\Delta f_o \gg 1$. It is interesting to note that $2\pi\Delta f_o$ may be interpreted as the rms phase modulation of the carrier $2\pi f_o$, and is related to the modulation index in sinusoidal (deterministic) frequency modulation. The two cases are thus for low and high "modulation indices".

In Figure 1 is presented the measurement methodology for test runs under these two distinct sets of conditions. The results of these two test runs are shown in Charts I and II. We will now describe the two spectra analytically, and compare the analytical predictions to the observations.

Case 1: $2\pi\Delta f_o < 1$, i.e. , $2\pi\Delta f_o = 1$ radian is a useful limit.

Then the output spectrum consists of
a) a sinusoid at f_o whose power is reduced by the factor $e^{-2\pi^2\Delta^2 f_o^2}$

i.e., $$\frac{S}{S_o} = \frac{\text{power in output sinusoid}}{\text{power in input sinusoid}} = e^{-2\pi^2\Delta^2 f_o^2} \quad (11)$$

A plot of (11) is presented in Figure 2.

b) A pair of symmetric sidebands with the shape of the spectrum of the integrated record/ play back flutter.

$$\boxed{\text{Sideband power spectral density} = 2\pi^2 f_o^2 \, G(f)} \tag{12}$$

Here Δ and G(f) are both intimately affected by the detailed shape of the spectrum of the flutter.

For a flat spectrum, it is seen that Φ_G is constant, and the spectrum of G(f) would be as shown in Figure 3.

For transports with a reasonably fast position servo, the low frequency flutter components are largely corrected by the servo action. For a typical machine, the correction capability is represented approximately by the transfer function

$$H(s) = \frac{s^2/\omega_T^2}{1 + 2\xi s/\omega_T + s^2/\omega_T^2} \tag{13}$$

where

s = Laplace operator.

ω_T = Radian natural bandwidth of servo.

î = Damping factor of servo ≈ .707.

Under conditions of a flat flutter spectrum input, the resultant spectrum of G(f) is described by

$$G(f) = \frac{\Phi_G}{(2\pi f_T)^2} \times \frac{f^4/f_T^4}{1 + f^4/f_T^4} \tag{14}$$

This is shown in Figure 4.

Chart I is a chart record of the spectrum of a 500 kHz sinusoid recorded and reproduced on a TICOR II phase locked to the 500 kHz sinusoid from the tape. The analyzer bandwidth is 6Hz. We will compare the results from the chart with those predicted from Equations (11) and (12), using published specifications for peak-to-peak flutter and peak-to-peak TBE. Since both of these variables are essentially stationary Gaussian random processes, we will assume that the rms value is 1/6 the peak-to-peak value, i.e. , we use the conventional 3 sigma approximation.

Example: Flutter = 0.4% pp

$$\sigma_F = \frac{4 \times 10^{-3}}{6}$$

$$\sigma_F^2 \cong 5 \times 10^{-7}$$

If the flutter is assumed to be essentially flat and concentrated in a 5 kHz band, (using (3)),

$$\Phi_G(f) = \frac{5 \times 10^{-7}}{5 \times 10^{3}} = 10^{-10}$$

If f = 300 Hz and f_o = 500 kHz

$$\text{sideband PSD} = \frac{2\pi^2 f_o^2}{4\pi^2 f^2} \Phi_G(f)$$

$$= \frac{1}{2} \left[\frac{500 \times 10^3}{300}\right]^2 \times 10^{-10}$$

$$= 1.4 \times 10^{-4}$$

$$\therefore PSD_{db}[@ f = 300] = 10 \log 1.4 \times 10^{-4} = -39 \text{ db}$$

Here we have picked f = 300 Hz = f_o for the servo involved, and thus are measuring at the peak of the flutter-induced sideband.

In a 6 Hz slot this would be

-39 db + 10 log 6 = -31 db (15)

The observed level at 300 Hz from the 500 kHz carrier (see Chart I) is

Observed level = -30 db (16)

Case 2: $2\pi\Delta f_o >> 1$

Here the output no longer contains a distinguishable sinusoid and consists of a pair of symmetric sidebands around the suppressed carrier. The power spectrum is Gaussian and the power spectral density is given by

$$PSD = \frac{S_o}{\sqrt{2\pi}\,\sigma_F f_o} e^{-\frac{f^2}{2\sigma_F^2 f_o^2}} \quad (17)$$

Both the level and the shape of this spectrum depend only upon the input power, the mean square total flutter, and the recorded carrier frequency. They are independent of the detailed shape of the flutter spectrum.

Chart II is a chart record of the spectrum of a 500 kHz sinusoid recorded and reproduced on a TICOR II. Here the machine is operated in the TACH mode, resulting in much higher TBE but somewhat less overall flutter. We will compare the results from the chart with those predicted by Equation (17), using published peak-to-peak flutter data.

Example: Flutter = 0.25%.

Here

$$\sigma_F^2 \cong 2 \times 10^{-7}$$

$$f_o = 500 \times 10^3$$

$$PSD \ @ \ (f = 0) = \frac{1}{\sqrt{2\pi} \cdot 4 \times 10^{-4} \cdot 500 \times 10^3}$$

$$= 1.8 \times 10^{-3}$$

$$PSD_{db} \ @ \ (f = 0) = -27 \text{ db}$$

Correcting for 6 Hz bandwidth, one would observe

$$-27 + 8 = -19 \text{ db} \qquad (18)$$

We do observe -20 db (Chart II). (19)

Since $\sigma_F^2 f_o^2 = (2 \times 10^{-7})(25 \times 10^{10})$

Then $\sigma_F^2 f_o^2 = 50 \times 10^3$

The plot of $10 \log \left[.01 \, e^{-\frac{f^2}{2 \times 50 \times 10^3}}\right]$

is shown superimposed upon Chart II. The reasonable fit is apparent.

Conclusions When a fundamental frequency f_o is recorded and reproduced, the resultant output spectrum may be described in terms of f_o and the flutter characteristics of the recording/reproducing system. The character of this spectrum is simply defined for most cases of practical interest, and the model presented correlates well with observation. An examination of the flutter characteristics of a potential machine, in light of these results, should allow the system designer to select an appropriate tape recorder/reproducer system to fit his data processing needs. Both the overall level and frequency distribution of the flutter play an important role in meeting severe spectral purity requirements, and the suitability of using a given machine for a specific requirement can be determined.

References

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5. Stewart, J. , The power spectrum of a carrier frequency-modulated by Gaussian noise, Proc. IRE, Vol. 42, Oct. 1954, pp 1539 - 1542.

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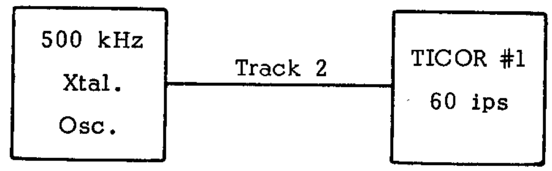
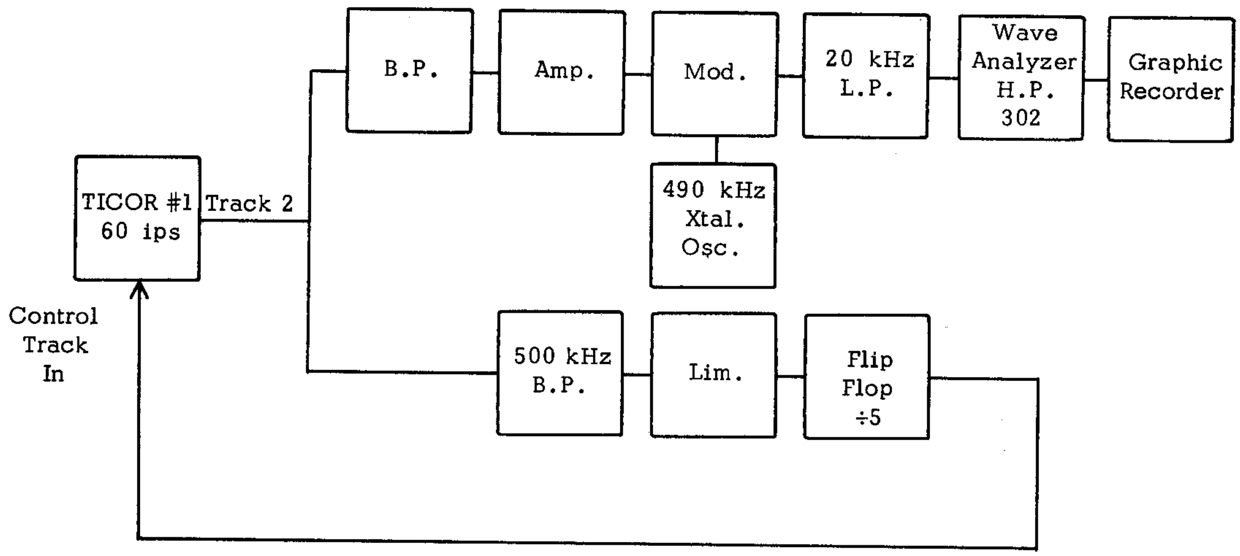


RECORD MODE



PLAYBACK MODE, TAPE LOCK

For TACH LOCK, tachometer is
locked to internal 100 kHz reference

Fig. 1-Measurement System
for Charts I and II

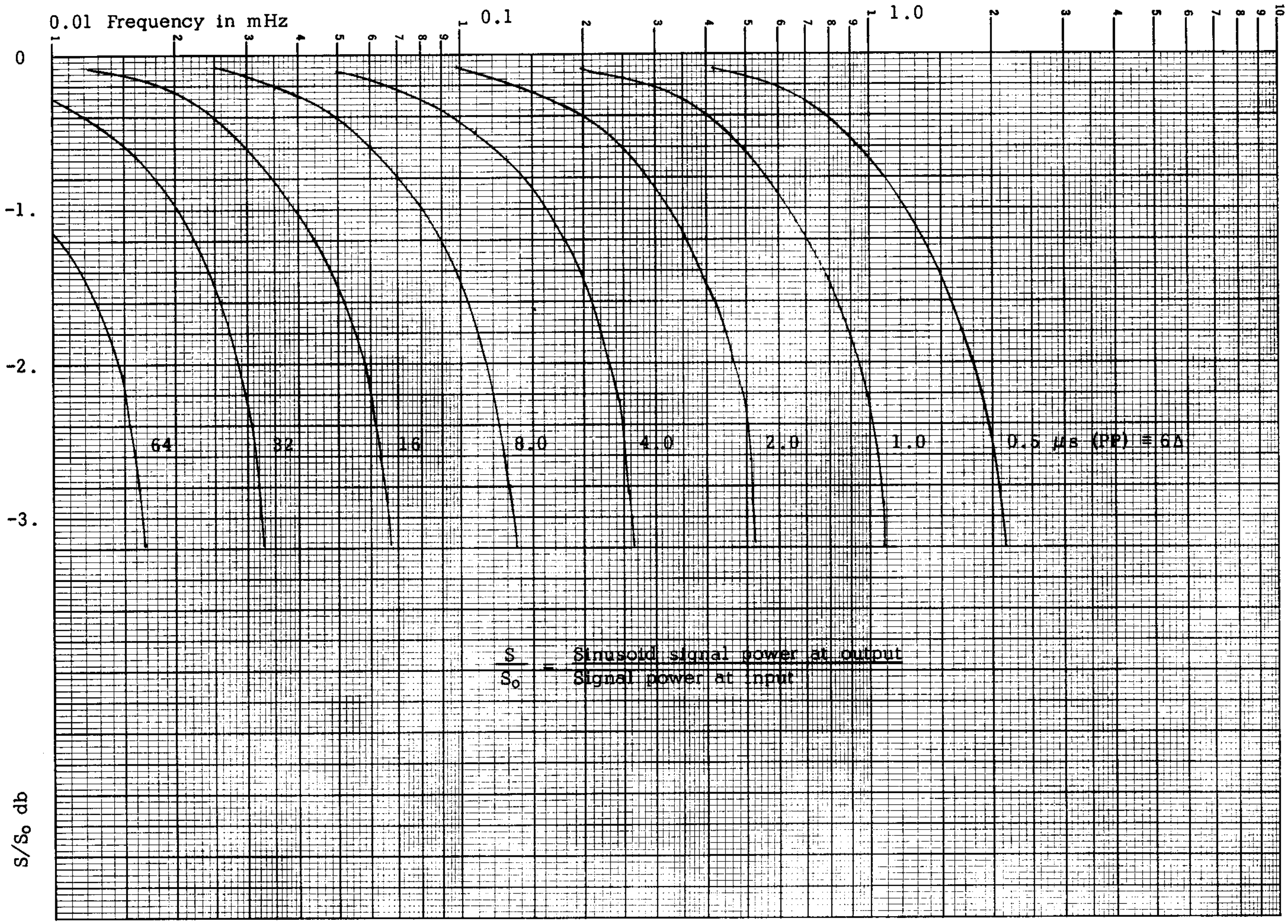


Fig. 2-Carrier Suppression Due to Flutter

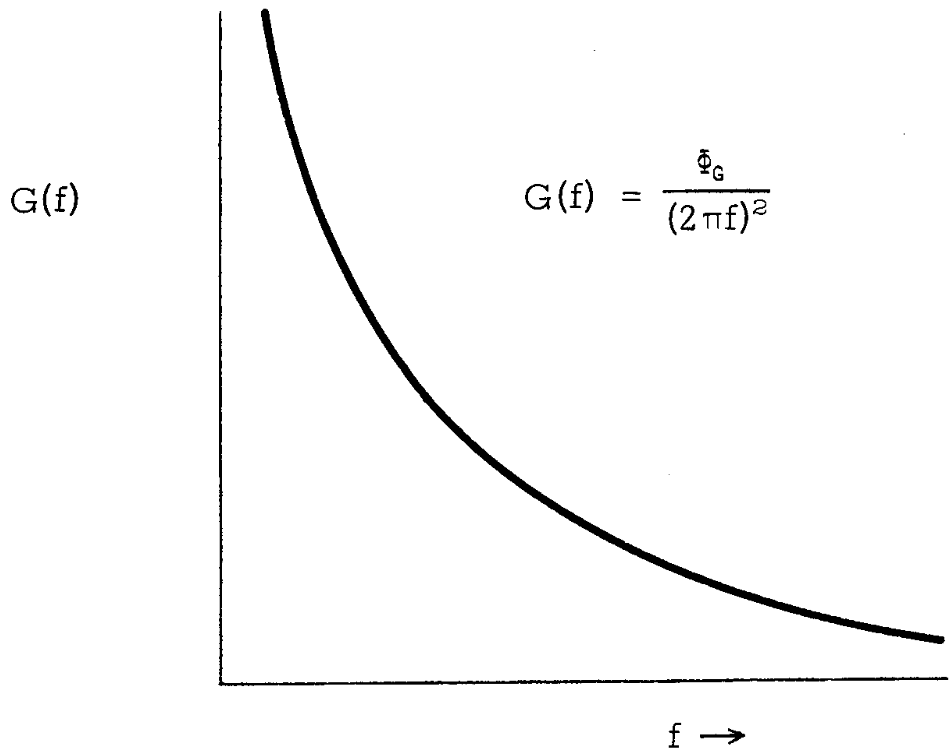


Fig. 3-Spectrum of Integrated Flutter

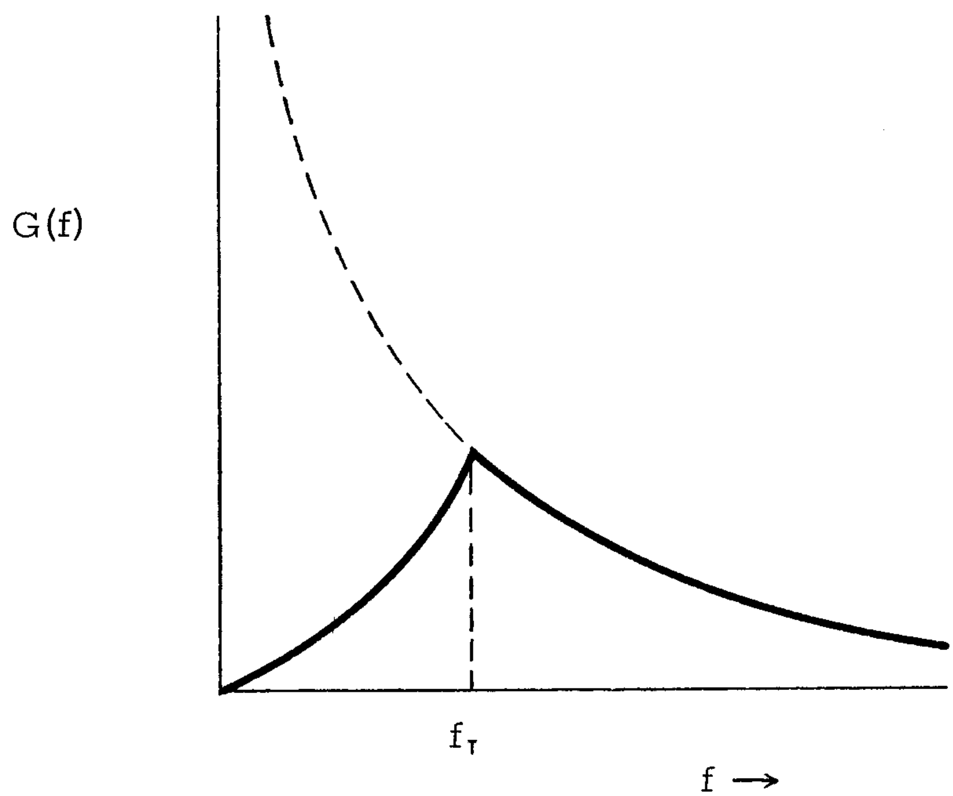


Fig. 4-Spectrum of Integrated Flutter With A Position Servo

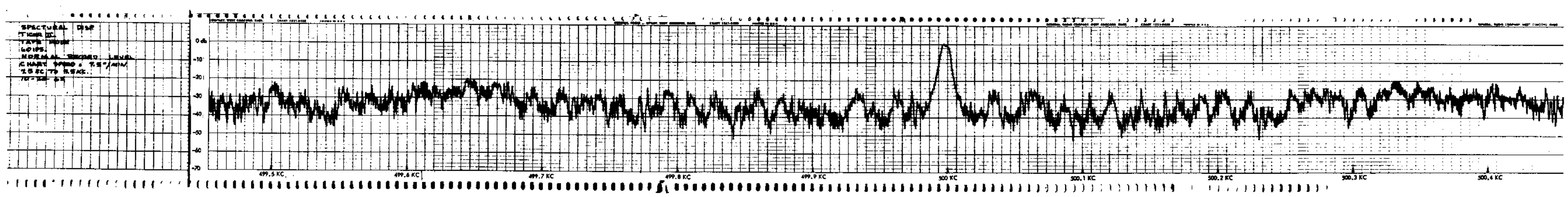


Chart I-Tape Mode
Taken in 6~ Slot

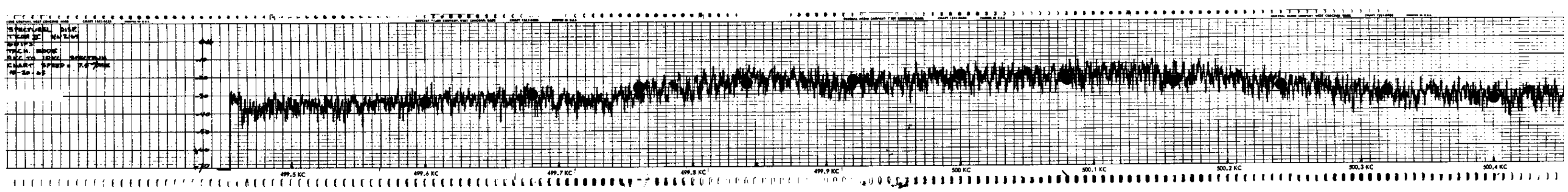


Chart II-Tach Mode
Taken in 6~ Slot