Comparison of Water Pricing Structures from a Collective Utility Viewpoint

by

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INTRODUCTION

As a result of continually lowering water tables in the arid regions of the West, researchers on water resource projects have begun realizing that it would be a good idea to consider water as any other rare resource and to let economic factors regulate its distribution (supply and demand). The control or balance of the economic factors is in the power of the public domain as represented by a city water management agency. This agency establishes some price rate structure for the collection of revenues. Metering is found to be a reliable, practical means of measuring the water consumption of users, and revenues are collected according to the measured consumption.

Historically there are four basic areas or categories of consumption. There are agricultural, industrial, commercial, and residential. Each category has its own rate structure. The revenue collected by the city water management agency then is a function of the category and the rates assessed in that category. In a study of public utilities, Whitford (1970) found that on the average of 41% of the consumed water occurred in the residential category.

24% in industrial, 18% in commercial, and 17% elsewhere. Thus, depending on the rate structure of the residential category, a good percentage of the revenue collected by the water agency occurs from residential consumption.

At present, Tucson's only source of water lies underground, and as Tucson's population grows, the pumping of water from this source will increase if no other source becomes available. The concern here is how a water agency can best accomplish the distribution of water to maximize benefits to the community. It may be desirable to decrease not only average consumption, but also peak consumption, which in Tucson, occurs during the summer because of sprinkling, evaporative coolers and swimming pools. To conserve water, the agency itself must practice a sound economic policy which is expressed as a price structure. We propose in this investigation a model for comparing two price structures from a collective utility standpoint in an effort to define a monetary value for the conservation of water.

TYPES OF PRICE STRUCTURES

There are basically three price structures for assessing water rates. One is called the flat rate. The flat rate system assesses upon the consumer a fixed price per billing period which is dependent upon many factors, e.g., type of dwelling (apartment, farmhouse), business establishment, or type of plumbing fixtures. Another structure is the step rate system. Under this system the unit rate is dependent on the total quantity consumed. The last type of price structure is the block rate. Under this system a new unit price is prescribed for each block of the total quantity consumed.

Each of these structures may be regressive or progressive. In a regressive pricing structure, rates decrease as quantity increases (a policy of marginal pricing and non-conservation). In a progressive pricing structure, rates increase as consumption increases (a policy of conservation). The City of Tucson presently uses a regressive block rate pricing structure.

This paper will consider a change of pricing structures for each of two periods from a collective utility standpoint. The collective utility is examined for a change in the present regressive rates to a permanent and seasonal progressive block rate pricing structure.

Before discussing collective utility in terms of a change in water pricing structure, we will try to present a brief theoretical overview of collective utility.

THE MODEL: THEORY

What is collective utility? Collective utility is an idealized measure of the worth of an economic state. We can only calculate the value of a change in collective utility which is then a measure of the value of a change from one economic state to another. We can consider four classes or sectors of the economy contributing to the economic state within jurisdiction of the agency. These four classes are residential, industrial, commercial, and agricultural. Each of these sectors is in an economic state and each can be measured in terms of collective utility.

For purposes of our mathematical model we define our variables with the following indices:

i	goods or services
k	persons, individuals
h	firms, companies
Q(i,k)	quantity of good or service of type i consumed by individual k
Note:	Q(i,o) is the initial quantity of i Q(i) is the total quantity consumed $Q(i) = \sum_k Q(i,k)$
Q(i,h)	quantity of good or service of type i produced by firm h
p(i,k)	price of good i paid or obtained by k
p(i,h)	price of good i paid or obtained by h
r(k)	revenue or income obtained by k
u(k)	real income of k
r(h)	revenue or income obtained by h

The satisfaction function of individual k is defined to be (Lesourne, 1964)

$$S(k) = S[k,Q(1,k), Q(2,k), \ldots, Q(I,k)] .$$

The production function of firm h is a function of the goods and services consumed and produced by the firm:

$$f(h) = f[h, Q(1,h), \ldots, Q(m,h)] = 0 .$$

This function, which may be vector valued, describes technological and other constraints.

An indifference curve for individual k describes the trade-off of good i

versus good j for a given value of satisfaction, S(k). Thus, individual k receives as much satisfaction by consuming x units of good i and y units of good j as by consuming x' units of i and y' units of j. A set of indifference curves for individual k may thus be defined by considering different values of satisfaction; each curve of constant satisfaction represents the trade-off of i for j. We assume that S(k) is constrained by k's income, r(k). If the society consists of two individuals, k and m, we may find that, by a proper redistribution of goods i and j, k may rise to a higher level of satisfaction, from $S_1(k)$ to $S_2(k)$, while m remains on his current indifference curve (satisfaction unchanged).

Extending this argument to a multi-dimensional concept, where each dimension represents a good, we may define an indifference hypersurface which gives the trade-off between all goods included in the satisfaction function of individual k.

An economic state can then be defined as the Cartesian product of the set of indifference hypersurfaces (one element per individual k).

The collective utility is a function of the indifference surfaces which in turn depend on the individual satisfaction functions.

$$U(S) = U(S(1), S(2), \ldots, S(k))$$

The collective utility function then provides a measure of the total flow of goods within a given economic state because each satisfaction function gives a relationship among these goods. As stated earlier, individual k's consumption is limited by r(k):

$$f(k) = \sum_i p(i)Q(i,k).$$

A change in collective utility, dU, then will measure the worth of the change from economic state 1 to 2. If dU is negative, the monetary value of the flow of goods is lessened; if dU is positive, the flow is increased relative to the initial state.

The basic expression for dU will be derived using the following working hypotheses.

H-1. For each fixed distribution of income, r(k), a system of prices and a set of quantities exist so that each individual's satisfaction is maximized. Each individual feels satisfied that he is getting the "most" for his money.

H-2. p(i,k) depends only on Q(i,k), i.e., the price of good i depends only on the demand-to-offer ratio of good i and is independent of the flow of the goods. There is no discount on the price of good i for buying good j.

H-3. Every individual pays the same price for a given good there is no favor in price to anyone.

$$p(i,k) = p(i)$$

H-4. $\Delta r(k)$ equals $\Delta u(k)$, i.e., with a proper selection of units a change in the measure of k's economic state equals a change in k's purchasing power.

H-5. Assume k's taste for good i remains constant, i.e., the desirability of good i remains the same. From these hypotheses and the relationships:

$$S(k) = S[k, Q(i,k)]$$

$$r(k) = \sum_{i} p(i)Q(i,k)$$

$$U = U(S(1), S(2), \ldots, S(k))$$

we find,

$$dU = \sum_{i,k} \left[p(i,k) + \frac{dp(i,k)}{dQ(i,k)} Q(i,k)\right] dQ(i,k)] \qquad (1)$$

U is assumed differentiable to yield dU. S(k) is maximized by Lagrange multipliers, constrained by r(k) and substituted into the equation for dU.

Normally it would be necessary to continue the theoretical developments to derive an expression for dQ(i,k). In the collective utility theory dQ(i,k) must account for the substitution of goods upon a change in the system of prices and set of quantitites. dQ(i,k) may be computed by use of the Slutsky coefficients, which, however, is beyond the scope of this paper.

THE MODEL: IMPLEMENTATION

The expressions for dU imply a knowledge of p(i), Q(i,k), dp(i) and dQ(i,k). The last variable is the most difficult to obtain and the most controversial to define accurately. Assumptions (indicated by the letter A followed by a numeral) will enable us to implement the model:

A-1 There are no goods to be substituted for water. Thus, dQ(i,k) (normally computed using the Slutsky coefficients) may come from empirical data of past changes or inferred from demand schedules. We have chosen the latter means. That is, dQ(i,k) can now be determined by the proper selection of a

value for the elasticity of demand n:

$$n = \frac{\Delta Q}{Q} / \left(\frac{\Delta p}{p}\right)$$

A-2 The value of n is the same for all individuals k.

A-3 The demand for water by industry is inelastic. Brown (1968) estimates that water costs account for about one percent of total production costs in industry, so water costs are relatively minor in minimizing total costs of production. This of course means that dQ(i,h) may be considered as having a small effect on conservation. Society can force industry's demand schedule toward elasticity by raising the price of water to a point where percentage of production costs by water becomes significant.

A-4 The demand schedule for commerce will be taken in first approximation as identical to that for industry.

Thus, in computing dU for purposes of studying conservation, we shall only be concerned with the residential sector of the community. This simplifying assumption will be removed in further studies to be published at latter date.

The Tucson Department of Water and Sewers is currently in the process of developing computer programs designed to estimate the revenue gained from water each month. Thus, there are some reliable data available, but at this time these data are not suited to our mathematical model, and are far from complete. Without having to go through each user's bill, it is possible to use the data from the computer print-out with some assumptions. A sample print-out is given

for one month for one of four metered areas (inside Tucson, outside Tucson, South Tucson, and remote or special areas). There are three rates according to consumption groups; the first is a flat rate while the succeeding two are incremental.

A-5 The distribution of bills within a consumption group is uniform. We expect to construct frequency distributions for each type of service for each area in the continuation of this research, but a uniform distribution allows us to demonstrate the applicability of the model although the results will be inaccurate.

A-6 The elasticity of demand is -.5 for annual residential use in Tucson. Howe and Lineweaver (1967) give a weighted mean of -.405, Clausen (1970) gives -0.35 while stating that the Tucson value is much higher, and Whitford (1970) gives -0.5 as weighted mean from a John Hopkins study for western areas.

EXAMPLE OF IMPLEMENTATION

For this example we will only consider the dU for the month of February, 1970, for $\frac{3}{4}$ in. service pipe (residential in the Remote area). Data are presented below.

Let

i = water rate 1, 2, or 3

$p(i)$ = price of water i in state 1

$dp(i)$ = change of price for water i

$Q(i,k)$ = k's consumption of water i in state 1.

$$dQ(i,k) = \frac{nQdp(i)}{p(i)}$$

$\frac{3}{4}$" Users Remote	Consumption	Rate	# Users
In state 1:	0 - 700 ft.^{3}	\$7.50	142
	800-3700 ft.^{3}	.40/100 ft.^{3}	425
	3800 - ft.^{3}	.40/100 ft.^{3}	30

Let us hypothesize 2 as follows:

0 - 700 ft.^{3}	\$7.50	
800-3700 ft.^{3}	.41/100 ft.^{3}	to be determined
3800 - ft.^{3}	.43/100 ft.^{3}	

In this problem the price paid for good i is not independent of the quantity consumed. Thus

$$\frac{dp(i)}{dQ(i,k)} \neq 0$$

and

$$dU = \sum_{i,k} [p(i) + \frac{dp(i)}{dQ(i,k)} Q(i,k)]\, dQ(i,k) \tag{2}$$

becomes

$$dU = \sum_{i,k} [p(i)dQ(i,k) + dp(i)Q(i,k)] \tag{3}$$

Since dp(i) is positive, dQ(i,k) is negative and an individual k in group i = 3 may, by reducing consumption, fall into i = 2. Thus, we must be careful to recognize this "shift" when performing the summation in (3).

Define H = {k|k is a user in the remote area of service type $\frac{3}{4}$"}

In state 1, H(1) = K1 + K2 + K3, with K1·K2·K3 = $\emptyset$;

for example, K2 = {k|k is a user in state 1 whose maximum consumption places him in rate i = 2}

In state 2, $H(2) = [K1 + \Delta K(2\text{-}1)] + [K2 + \Delta K(3\text{-}2) - \Delta K(2\text{-}1)]$

$$+ [K3 - \Delta K(3\text{-}2)]$$

where e.g., $K(2\text{-}1) = \{k | k$ is a user who shifts from rate $i = 2$

to rate $i = 1\}$

Now let us set up the dU formula with the proper summation sets

$$dU = \sum_{\substack{i=1 \\ k \epsilon H(2)}}^{3} [p(i)dQ(i,k) + Q(i,k)dp(i)]$$

for $i = 1$,

$$dU(1) = \sum_{k \epsilon K1 + K2 + K3 = h} [p(1)dQ(1,k) + Q(1,k)dp(1)]$$

(everyone pays $i = 1$)

$$dU(1) = Hdp(1)$$

since all k's must pay dp(1), dQ(1,k) is irrelevant because p(1) is a flat rate. Also there is no change hypothesized for $i = 1$ so, $dU(1) = 0$, i.e., there is no change in collective utility for $i = 1$. Now for $i = 2$

$$dU(2) = \sum_{k \epsilon K2 + \Delta K(3\text{-}2) - \Delta K(2\text{-}1)} [p(2)dQ(2,k) + Q(2,k)dp(2)]$$

$$+ \sum_{k \epsilon K3 - \Delta K(3\text{-}2)} [p(2)dQ(2,k) + Q_m(2,k)dp(2)]$$

where $Q_m(2,k)$ is the maximum consumable quantity in rate $i = 2$ for k in rate $i = 3$; also, $dQ(2,k) = 0$ for k in rate $i = 3$, therefore,

$$dU(2) = \sum_{k \epsilon K2 + \Delta K(3\text{-}2) - \Delta K(2\text{-}1)} [p(2)dQ(2,k) + Q(2,k)dp(2)]$$

$$+ \sum_{K \epsilon K3 - \Delta K(3\text{-}2)} Q_m(2,k)dp(2) \qquad (4)$$

now for i = 3

$$dU(3) = \sum_{k \epsilon K3 - \Delta K(3-2)} [p(3)dQ(3,k) + Q(3,k)dp(3)] \quad (5)$$

To determine the ΔK sets, use A-2 and A-6 to obtain

$$\Delta Q = \frac{nQ\Delta p}{p} = \frac{(-.5)\ (41)\ (.03)}{.4} = 1.54$$

Thus each k paying rate i = 3 will reduce his consumption by an average of 154 ft.^{3}.

Using A-6,

$$\Delta K(3\text{-}2)(26.7)\text{ft}^3/\text{user} = 154\ \text{ft.}^3, \text{ yielding } K(3\text{-}2) = 5.$$

Thus of the k's paying rate i = 3 there will be 5 who shift from rate i = 3 to rate i = 2.

We can find by the same method all quantities in the above formulas. The computations are, for evaluating dU, found in Appendix A. The results are:

PERMANENT RATE CHANGE

1.	$[\Delta Q(2)] = 25\ \text{ft.}^3$	$[\Delta Q(3)] = -154\ \text{ft.}^3$
2.	$dU(2) = \$24.63$	$dU(3) = -\$12.00$
3.	$K(2\text{-}1) = 4$	$\Delta K(3\text{-}2) = 5$
4.	Total dU = $12.63	
5.	Total dU for 12 months = $150	

Thus, if November is considered as an average month, the annual increase in collective utility would be about $150 for the Remote Area of service type $\frac{3}{4}$". Thus, the rate change would be recommended.

SEASONAL RATE CHANGE

Now let us examine the change in collective utility if the price rate structure is changed during the summer and remains as now during the winter.

We will make the same assumptions as for the preceding example and select the month of September as the sample month for the summer. Therefore, for the Remote Area of service type $\frac{3}{4}$", the following rates are to be charged during the six summer months:

Type	Consumption	# Users	Rate	
			State 1	State 2
$\frac{3}{4}$"	0 - 700 ft.^{3}	106	$7.50	$7.50
	800-3700 ft.^{3}	436	.40/100 ft.^{3}	.41/100 ft.^{3}
	3800 - ft.^{3}	41	.41/100 ft.^{3}	.43/100 ft.^{3}

The elasticity of demand during the summer will be taken to be -.7 (Howe and Linaweaver, 1967). For the summer then the results are as follows:

1. Q(2) = -38.5 ft.^{3} Q(3) = -251 ft.^{3}
2. dU(2) = $4.63 dU(3) = -$16.25
3. K(2-1) = 5 K(3-2) = 8
4. Total monthly dU = -$11.62
5. dU for 6 months = -$70

Thus a price rate structure changed only for the summer would apparently be detrimental to the economic state of the area in question. A closer scrutiny of such a seaonal rate change shows that it is still beneficial.

COMPUTATION OF ΔQ AND ΔR

Why is there a difference in the sign of dU for the proposed rate changes? To answer this question we will compute the change in revenue and the change in water consumption for both the permanent change and the seasonal change. In computing ΔR and ΔQ for both cases we use the preceding assumptions given in finding dU.

Because there is no change in the first rate, there will be no noteworthy changes in the consumption pattern of those users who only have to pay the first rate.

(a) For the permanent change,

$$\Delta R = \$216 \text{ for 12 months}$$

i.e., The revenue is increased by the change; and

$$\Delta Q = -200{,}000 \text{ ft.}^3 \text{ for 12 months}$$

i.e., water was conserved.

(b) For the seasonal change,

$$\Delta R = -136{,}800 \text{ ft.}^3 \text{ per season}$$

i.e., water would be conserved.

In tabular form,

Permanent Change	Seasonal Change
$\Delta R = \$216$	$\Delta R = -\$31.32$
$\Delta Q = -200{,}000 \text{ ft.}^3$	$\Delta Q = -136{,}800 \text{ ft.}^3$
$n = -.5$	$n = -.7$
$dU = \$150$	$du = -\$70$

Note that the signs of ΔR and dU are the same.

The computer was used to perform further tedious numerical calculations. Under our assumptions the following values for the month of November were obtained:

n = -.5	dU(2)$	dU(3)$	ΔR$	ΔQ/100 ft.^{3}
Remote $\frac{3}{4}$ in.	24.63	-12.00	18.32	-163
South $\frac{3}{4}$ in.	41.91	-28.16	-5.04	-431
Outside $\frac{3}{4}$ in.	138.09	-59.47	129.43	-1186
Inside $\frac{3}{4}$ in.	1593.55	-888.13	-896.26	-32671
	Total dU		Total ΔR	Total ΔQ
	810.43		-753.55	-34451

For the month of September:

n = −.7	dU(2)$	dU(3)$	ΔR$	ΔQ/100 ft.^{3}
Remote $\frac{3}{4}$ in.	4.63	-16.25	-5.22	-228
South $\frac{3}{4}$ in.	2.69	-32.94	-60	-603
Outside $\frac{3}{4}$ in.	-1.59	-73.66	-12.33	-1660
Inside $\frac{3}{4}$ in.	-1215.38	-257.18	-3714.11	-45740
	Total dU		Total ΔR	Total ΔQ
	-1589.68		-3791.65	-48232

DISCUSSION: RESULTS

With either suggested price structure the water utility agency under our hypotheses will lose money and water consumption will be lowered. However, under the permanent change the monetary value of the flow of goods would be

be relatively greater while under the seasonal structure of the flow of goods would be relatively decreased. In economics the flow of goods indicates the "health" of the economy. Thus, under our hypotheses the permanent structure change would be recommended and the temporary would not be suggested. If it is deemed by the public to conserve water, then the temporary structure could be implemented although the "health" of the economy might suffer.

The reader might expect that $\Delta R = dU$, since the flow of goods exists only between the agency and the individuals. The value for ΔR and dU are different because ΔR is a function of the consumption levels and prices in State 1 versus State 2, whereas dU is a function of the consumption and price changes given the State 1 values. Since the price is increased in our example, Qdp gives a value for the flow when the consumption remains constant and the price remains constant and the price is changed, and pdQ gives a value for the flow when the price remains constant and the consumption changes. dU then reflects a trade-off between these two flows. A negative value of dU implies that pdQ is larger than Qdp; that is, the reduced consumption more than offsets the increase in price, thus water is conserved.

The phenomenon of shifting accounts for the difference between ΔR and dU. ΔR is computed post facto as a function of the state of nature and unit prices. dU, on the other hand, is computed on a marginal basis and so reflects the change of flow as a function of these marginal values and the given state of nature.

In the arid regions of the Southwest where water is among the scarcest resources, we see that dU is a better measure of a change in pricing than

ΔR. We note that it is possible to have a negative ΔR (indicating conservation on the basis of a loss in revenue) and a positive dU. The difference between ΔR and dU can be enlarged if the price break between rates is increased. Furthermore, industry and commerce can contribute nothing to conservation until society raises their elasticity to a point where the costs of water become a significant proportion of production costs.

If the agency adopts a policy of increasing the water rates on a programmed basis, collective utility provides a mechanism for considering the substitution of goods, e.g., refrigerated cooling instead of evaporative cooling and non-water consumptive forms of landscaping instead of lawns, shrubs, and trees.

Several criticisms can be raised about this model. The first concerns the abundance of assumptions. Perhaps the most significant assumption relating to implementation concerns a uniform distribution of users within a group because it forces us to define an upper limit in the third consumption group (in this case 4500 $ft.^3$). We are taking steps to construct a frequency distribution function which will obviate the need for the assumption of uniformity Once we have constructed a frequency distribution function we can consider the influence of commercial and industrial consumption.

The assumption of applying the same demand schedule to all individuals is plausible after considering certain factors. Those individuals who consume the most generally can afford to pay according to their consumption. Furthermore, few individuals know to which price rate group they belong; so no individual will lower his consumption more than the demand schedule indicates. Other elasticities could easily be considered according to the economic sector of the city in which the individuals live.

Higher rate change structures could be hypothesized to find which structure causes the value of a positive dU to change to a negative dU. Under our assumptions such changes in structure have been hypothesized and a sensitivity analysis conducted with the aid of a computerized model.

For this particular model we changed the limit in the third rate to 6100 ft.^{3}. We made two runs in computing dU and ΔR. The first run was on the present regressive rate structure with no increase in price for rate i = 1, a 0 to 50% increase within price for i = 2, and a 0 to 50% increase in price for i = 3 both in increments of 5% for each of the four billing areas in Tucson. This was repeated for elasticities from -.3 to -.65 in increments of -.05. The second run was on a progressive rate structure with no increase in price for rate i = 1, a 0 to 50% increase in increments of 5% for i = 2, and a 0 to 100% increase in increments of 10% for i = 3 for each of the four billing areas in Tucson. This also was repeated for elasticities from -.3 to -.65 in increments of -.05 . For both runs it was found that dU started with a positive value, increased to a maximum and then decreased for all elasticities except -.65. At -.65, for the regressive structure, dU started positive and decreased for each increase; for the progressive structure, dU started negative and decreased for each increase in price. The sensitivity analysis also shows that, for a given elasticity and level of percentage increase, dU for the progressive structure is always less than dU for the regressive structure, the difference being due to the greater shifting from i = 3 to i = 2 in the progressive structure. However, dU is seldom negative except at the higher values of elasticity and at the greater changes in price which are politically impractical. The analysis does substantiate the belief that a progressive price rate structure

does encourage lower consumption of water than a regressive structure. It should be emphasized here that in all cases ΔR continued to increase in value well after dU had reached its maximum, i.e., the values of dU are more sensitive to shifting than the values of ΔR. The analysis also tends to indicate that once the community is on a progressive rate structure, the rates of $i = 2$ should be increased as much as the rates in $i = 3$ since the bulk of the residential consumers lies in $i = 2$. The consumer shift from group $i = 2$ to $i = 1$ would have even more impact on dU than the shift from $i = 3$ to $i = 2$. These results are illustrated in Figure 1 for a progressive pricing structure and in Figure 2 for a regressive one. The numbers labeling the lines represent the percentage of price increase in group $i = 1, 2, 3$, respectively.

CONCLUSIONS

The mathematical model for collective utility is intrinsically suited to a computer program. With the aid of the computer the user sets can easily be defined and a distribution function devised from available data. Such function could give the ΔK sets upon computation of ΔQ from a given n. These ΔK sets and distribution functions would of course give a more accurate value to the shift of users from one rate to another and lead to an accurate evaluation of a city-wide dU. Such a value of dU would enable the agency to evaluate the benefit of change of price rate when having to consider in their decision the benefit to the community of water conservation and the lowering of consumption peaks.

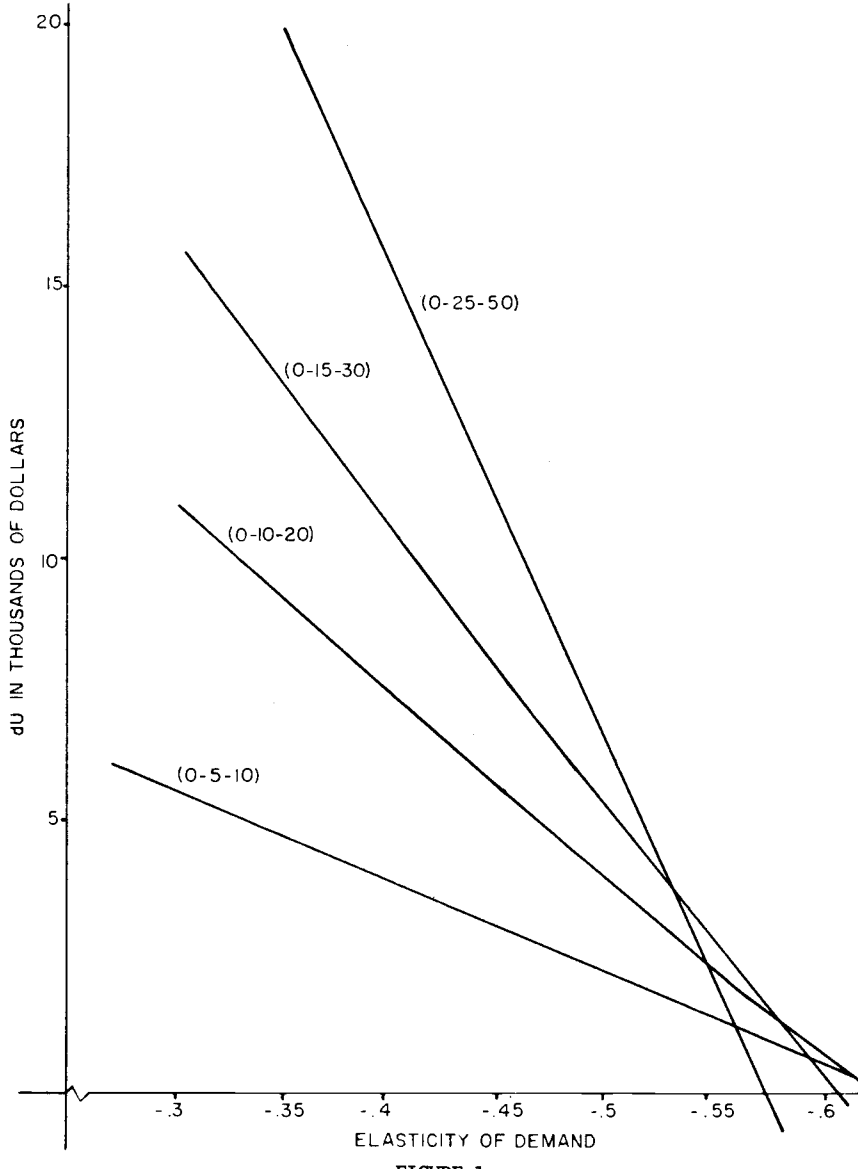

FIGURE 1

Collective utility change dU versus elasticity and price increase schedule for a Progressive Pricing Policy

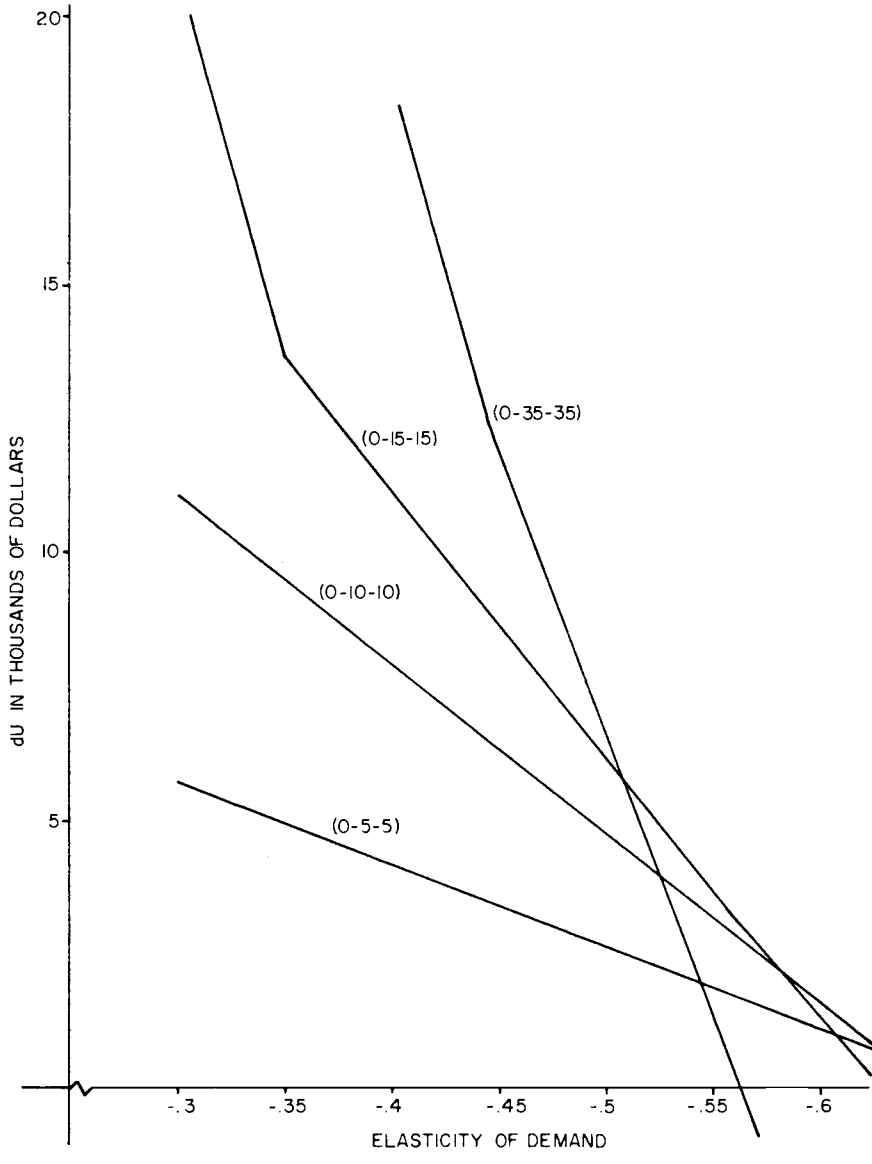

FIGURE 2

Collective utility change dU versus elasticity and price increase schedule for a Regressive Pricing Policy

APPENDIX A:

Numerical Evaluation of dU

We have shown earlier that dU for i = 1 is zero. For i = 2 then by A-5:

$$\frac{(3700 - 700)\ \text{ft.}^3}{425} = 7\ \text{ft.}^3/\text{user}$$

$$E[Q(2)] = 2200\ \text{ft.}^3$$

by A-5 and A-6 $E[\Delta Q] = \frac{n\bar{Q}\Delta p}{p} = \frac{(-.5)\ (22)\ (.01)}{.40} = 27.5\ \text{ft.}^3$

i.e., each k in state 1 in rate i = 2 will reduce his consumption by 27.5 ft.^{3} (this is on the average). By A-5 $\Delta K(2\text{-}1)$ (7 ft.^{3}/user) = 27.5 thus K = 4, i.e., there are k = 4 who will, by reducing consumption no longer have to pay rate i = 2. There are 4 k's in K(2-1). As computed in the main body, $\Delta K(3\text{-}2)$ = 5.7, by (3).

$$\begin{aligned} dU(2) = & \sum_{k\epsilon K2} p(2)dQ(2,k) + Q(2,k)dp(2) \\ & + \sum_{k\epsilon \Delta K(3-2)} p(2)dQ(2,k) + Q(2,k)dp(2) \\ & - \sum_{k\epsilon \Delta K(2-1)} [p(2)dQ(2,k) + Q(2,k)dp(2)] \\ & + \sum_{k\epsilon K3} Q_m(2,k)dp(2) - \sum_{k\epsilon \Delta K(3-2)} Q_m(2,k)dp(2) \end{aligned}$$

$$\begin{aligned} = & (.4/100)(-27.5)(425) + (2200-700)(.01/100)(425) \\ & + (.4/100)(-153.75/2)(5.7) + (3700-700)(.01/100)(5.7) \\ & - (.4/100)(-27.5)(4) - (2200-700)(.01/100)(4) \end{aligned}$$

$$+ (3700-700)(.01/100)(30-4)$$

$$= -46.75 + 63.75 - 1.773 + 1.73$$

$$+ .428 - .5835 + 7.83$$

$$= \$24.63$$

by (4)

$$dU(3) = \sum_{k\epsilon K3} p(3)dQ(3,k) + Q(3,k)dp(3)$$

$$- \sum_{k\epsilon \Delta K(3-2)} [p(3)dQ(3,k) + Q(3,k)dp(3)]$$

$$= (.4/100)(-153.75)(30) + (4100-3700)(.03/100)(30)$$

$$- (.4/100)(-153.75)(5.7) - (4100-3700)(.03/100)(5.7)$$

$$= -18.45 + 3.60 + 3.54 - .69$$

$$= -\$12.00$$

$$dU(1) + dU(2) + dU(3) = \$12.63$$

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