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THE EFFECTS OF INFLATION AND BUSINESS INCOME TAXES ON INVESTMENT AND NATURAL RESOURCE UTILIZATION

The University of Arizona

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THE EFFECTS OF INFLATION AND BUSINESS INCOME TAXES
ON INVESTMENT AND NATURAL RESOURCE UTILIZATION

by

Frank Andrews Mayne

A Dissertation Submitted to the Faculty of the
BUSINESS ADMINISTRATION COMMITTEE
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1982

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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Frank Andrews Mayne entitled THE EFFECTS OF INFLATION AND BUSINESS INCOME TAXES ON INVESTMENT AND NATURAL RESOURCE UTILIZATION and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of DOCTOR OF PHILOSOPHY.

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The combination of price inflation and historical cost-based depreciation for tax purposes has been shown in the finance literature to reduce the present value of depreciation deductions. Since tax depreciation deductions are limited to a nominal dollar amount, when inflation occurs a future tax deduction has less real value. This effect is pedagogically presented in a capital budgeting context.

In the economic literature Hotelling and Herfindahl have contributed models describing natural resource production changes in response to changes in demand, production cost, and cost of capital.

Parts of the inflation-explicit capital budgeting model and the Hotelling and Herfindahl models are combined. The result is a partial equilibrium model which yields the conclusion that production from a natural resource is reduced if inflation is increased.

Copper industry data from 1947-1978 are examined for empirical confirmation of the theoretical model. A copper industry production function, which contained demand, real price and labor strike variables, gained in descriptive capability by inclusion of inflation as an additional multiple regression variable.

The dissertation reviews literature on the relationship between capital investment and inflation. The cases of hyper-inflation in Germany and medium inflation in Latin America are considered. Other literature review topics include the inflation
effects in the securities markets, inflation-caused wealth transfers, and inflation adjustments in accounting statements.
CHAPTER 1

INTRODUCTION

This study focuses on the real effects on investment and natural resource utilization caused by the interrelationship of historical cost based income taxes and inflation. Historical cost based income taxes use historical cost based depreciation charges which decline in real value when inflation occurs. Then, a real increase in the cost of capital will occur, causing various capital budgeting effects.

Some of the capital budgeting effects which have been elaborated in the literature include favoring capital projects with comparatively shorter lives, deferring replacement of inefficient equipment, and the adjustment of the capital/labor ratio toward labor intensive alternatives (Nelson, 1976b).

Another effect which has not been fully developed in the literature is the relationship between production and inflation. This study will explore that relationship on both an empirical and theoretical basis, in the context of natural resource exploitation.

These capital budgeting effects are based on the theoretical premise that the customary discount rate factor used in capital budgeting should also be weighted by an explicit inflation factor. Then, when inflation rates are larger versus smaller, cash flows received in the future will be worth comparatively less present
value. The theoretical models in this study will show that the tax savings resulting from depreciation charges will have comparatively less present value with higher inflation rates.

The reason for the focus on the historical cost based depreciation charge is that it and it alone, cannot rise with inflation. In Chapter 2 a basic capital budgeting model will be presented which illustrates this point and provides a theoretical framework for use with subsequent discussion.

Chapter 3 will review literature which is relevant to the topic of the relationship of inflation and capital investment trends. Historical, theoretical, and empirical literature will be reviewed.

Chapter 4 will review literature on natural resource economics. It will emphasize theoretical models in the economics literature which will be essential to development of the hypothesis that inflation will affect the rate of exploitation of a natural resource.

Chapter 5 will present a theoretical model which asserts that there is a demonstratable negative relationship between inflation and the quantity of production from a natural resource. The model will be derived by combining the inflation-explicit capital budgeting model from Chapter 2 with various excerpts from the natural resource literature which was reviewed in Chapter 4.

An empirical study of the copper industry will be presented in Chapter 6. It will test whether or not the predicted relationship between production and inflation is present. The empirical study uses a multiple linear regression model in which an existing copper industry production function will have inflation added to the independent
variables in the model to see if the predictability of the model is improved with the inflation factor.
CHAPTER 2

THE DEPRECIATION-INFLATION-TAXES EFFECT IN CAPITAL BUDGETING

The capital budgeting model which illustrates the effect of inflation on the present values of future depreciation tax deductions is a fairly well discussed topic in the finance literature. Discussion of that topic includes articles by Bierman and Smidt (1975), Hong (1977a), Cooley, Roenfeldt, and Chew (1975), Cooley et al. (1976), Nelson (1976b), Gramlich (1976), Tideman and Tucker (1976), Terborgh (1960), and Raiborn and Ratcliffe (1979). Each of these discuss the concept that historical cost based income taxes cause returns and present values in capital budgeting to be inversely related to inflation rates. Each of these articles highlights the fact that the historical cost based income taxes limit the tax deduction for depreciation to the original cost of the investment, which in turn causes the real value of the cash flows resulting from the depreciation charges to decrease as the inflation rate is increased. It is from the background of the models presented by the above referenced authors, that the model in this chapter will be developed. It is not intended that this model is innovative, but rather it is intended that the model will be useful in some innovative material in subsequent chapters.

A historical cost based depreciation charge cannot rise with inflation. Nominal historical cost must be used. Other cash flow calculation variables such as prices and wages can be assumed to
increase proportionately with inflation, yet the historical cost tax system requires that only the original nominal amount of the investment can be depreciated.

It is important to note that the problem stems from an inter-relationship of taxes and inflation. Without an income tax, depreciation charges would have no present value; without inflation, the difference between nominal cost and real value would not exist. This point will be made mathematically at the appropriate point as the model is developed.

Some preliminary clarifications are in order before proceeding. It is a continuing topic of interest in the finance literature whether or not the capital structure of a given firm will influence valuation and investment policy. It is generally now conceded that there will be some influence from capital structure when either income taxes or imperfect markets are involved (Haley and Schall, 1973, pp. 215-32, 283-91; Van Horne, 1968, Chapter 7; Modigliani and Miller, 1958).

However, financing policy will not be relevant in the model which is presented in this chapter because it can be shown that the capital structure is not relevant to the present value of the depreciation charges. This is because the risk-free rate should be used in capitalizing the present value of the depreciation charge, while capital structure adjustment factors in investment policy models are used to adjust the risk present in model components other than the present value of the depreciation charge. The logic inherent in using the risk-free rate is based on the fact that specific dollar amounts of depreciation charges are usually always known with certainty since the
amount and timing of the depreciation is nearly always known with certainty at the time that the investment is made. The only situations where specific dollar amounts of cash flows from depreciation would be uncertain is when there are no tax carrybacks available, and future earnings are also uncertain. Further elaboration of the point is made by Haley and Schall (1973, pp. 171-75) or Cooley, Roenfeldt, and Chew (1975, pp. 23-24).

Based on the foregoing logic, the assumptions being made here can be stated as follows:

1. Capital structure is to be ignored in the model. This implies equivalence to a 100% equity case in a model which has a capital structure included as a relevant variable.

2. Risk is to be ignored. In effect, this is equivalent to 100% certainty of returns special case of a model in which risk is an included variable. This assumption also means that the model will not contain either risk adjusted rates of return or the covariance factor which represents adjustment for the variability of returns.\(^1\) In view of the previously stated fact that depreciation charges are usually certain, inclusion of either risk-adjusted rates of return or covariance corrections

\(^1\)For example, the covariance factor "\(\text{Cov}(X, r_m)\)" in \(\frac{X - \lambda \text{Cov}(X, F_m)}{(1 + i)}\) represents the fact that the risk adjustment for the expected value of cash flow \(X\) may vary from return \(r_m\) for the market as a whole. The tildes represent probability distributions of \(X\) and \(r\). \((1 + i)\) represents the discount rate. \(\lambda\) represents the slope of the capital market line. Elaboration is presented by Haley and Schall (1973, esp. Chapter 8).
for other cash flow variables would serve no useful purpose
and would make the model more cumbersome.

3. Initially, inflation will be assumed to be actual (or
anticipated inflation in the model. Uncertain inflation
will be introduced at an appropriate time.

The starting point for development of the inflation explicit
model will be the traditional discounted cash flow models, specifically
the one-period statements which serve to add income taxes to capital
budgeting. These models will be patterned after the discussion by

First, consider a model for the after tax flows received in
one particular year \( j \): Let

\[
[2.1] \quad \text{ATP}_j = (1-t)(R_j - E_j - D_j) + D_j
\]

or

\[
[2.2] \quad \text{ATP}_j = (1-t)(R_j - E_j) + tD_j
\]

where

\[
\text{ATP}_j = \text{Net After Tax Proceeds (or cash flow) for year } j;
\]

\[
t = \text{tax rate, assumed to be the same for all years;}
\]

\[
R_j = \text{cash flow revenues (on tax return) for year } j;
\]

\[
E_j = \text{cash flow expenses (on tax return) for year } j;
\]

\[
D_j = \text{depreciation (non-cash flow expense) for year } j.
\]

The two equations are algebraically equivalent. Consider
multiplying in equation [2.1] the \((1-t)\) factor by \((R_j - E_j - D_j)\) and then
collecting \(D_j\) terms.
Only tax return revenues and expenses are in these equations. Also, they are only one-period models. Discounting, multiperiod cash flows and investment outlays will not be added to the models. The following symbols will be used:

\[ V = \text{Net Present Value}; \]
\[ X_j = \text{Cash flow of period } j; \]
\[ n = \text{life of the investment}; \]
\[ k = \text{cost of capital to the firm}; \]
\[ r = \text{the internal rate of return of the investment}. \]

The "net present value" model (before considering taxes) can be expressed algebraically as follows:

\[ [2.3] \quad V = \sum_{j=0}^{n} X_j (1+k)^{-j} \]

The complementary internal rate of return (yield) model is expressed as follows:

\[ [2.3] \quad \sum_{j=0}^{n} X_j (1+r)^{-j} = 0 \]

The internal rate of return is computed by solving the equation for \( r \).

A net present value model is now suggested for multiperiod cash flows involving income taxes. The concepts expressed in [1.2]
and [2.3] above are combined, and an additional algebraic term, \( G_j \), is added to represent investment cash flows at time \( j \):

\[
[2.5] \quad V = \sum_{j=0}^{n} \frac{G_j}{(1+k)^j} + \sum_{j=0}^{n} \frac{(1-t)(R_j - E_j)}{(1+k)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+k)^j}
\]

The first term to the right of the equal sign often represents only an initial outlay at time 0, however the summation of years approach allows for a multiperiod investment situation.

The second term on the right represents cash revenues and cash expenses after both taxes and discounting. The third term represents non-cash expenses (and revenues where applicable), principally depreciation, which provide income tax savings since they are used in the tax computation as deductions.

The internal rate of return model can be stated as follows by using equations [2.2] and [2.4]:

\[
[2.6] \quad \sum_{j=0}^{n} \frac{G_j}{(1+r)^j} + \sum_{j=0}^{n} \frac{(1-5)(R_j - E_j)}{(1+r)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+r)^j} = 0
\]

The first term on the left is the discounted value of investment cash outflows, the second term is the summation of cash revenues and expenses entered into the tax computation, and the third term is the depreciation tax shield. As in equation [2.4] the equation is solved for \( r \) to obtain the internal rate of return.

Before adding an inflation factor to these discounted cash flow models, it is necessary to differentiate between nominal and real discount rates. When inflation exists, by necessity, both real and
nominal discount rates must exist. For example, if an investment

grows at a nominal rate \( h \) during period \( j \), when inflation rate \( f \) is

present, then the real rate of return will be \( r \) as in the following
equation (Reilly, Marquardt, and Price, 1977, pp. 2-3):

\[
[2.7] \quad r = \frac{1 + h}{1 + f} - 1
\]

In other words, the fact that in constant purchasing power, \((1+h)\)
dollars held at the end of period \( j \) representing principal and nominal
interest at rate \( h \) on one dollar must also reflect that \((1+f)\) dollars
are required to purchase what one dollar would have purchased at the
start of the period.

An algebraic rearrangement of the same equation produces the
following tautological relationship between the real and nominal rates
of return:

\[
[2.8] \quad \frac{1 + h}{1 + f} - 1 = r
\]

\[
[2.9] \quad \frac{1 + h}{1 + f} = 1 + r
\]

\[
[2.10] \quad (1+h) = (1+i)(1+f)
\]

The classic work on this relationship was developed by Irving
Fisher (1896, pp. 1-31, 50). A further elaboration is in Appendix A.

The last line above is especially applicable in this study.
It will reappear in models which will now be developed in this chapter.
Initially, add a simplifying assumption that the project has all cash outflows for investment ($G_j$) occurring at time 0. The model from (V) above reduces to:

\[
V = \frac{G_0}{(1+k)^0} + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)}{(1+k)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+k)^j}
\]

Now assume that cash revenues and expenses in subsequent years are subject to inflation rate $f$. In other words, the firm can raise prices, and in turn experiences increases in expenses, at the inflation rate. Then

\[
V = \frac{G_0}{(1+k)^0} + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)(1+f)^j}{(1+k)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+k)^j}
\]

By measuring present value in current dollars, a higher present value would result if the computation was made in this form. Yet the higher present value is clearly illusory because the dollars received at the end of the period have shrunk in terms of constant purchasing power, and the inflation index factor $(1+f)^j$ must be included in the denominator to restate the calculation in constant dollars:

\[
V = \frac{G_0}{(1+k)^0(1+f)^0} + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)(1+f)^j}{(1+k)^j(1+f)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+k)^j(1+f)^j}
\]
Simplifying and cancelling:

\[ V = - G_0 + \sum_{j=0}^{n} \frac{(1-t)(R_j - E_j)}{(1+k)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+k)^j(1+f)^j} \]

The denominator of the third right hand term has taken the form of the Fisher equation for the relationship between nominal and real rates. The other terms would have also been in this form except for simplifying and cancelling. The product of the factors \((1+k)^j(1+f)^j\) is the nominal discount rate which would be observable in current dollar terms in securities market data.

Also, in real terms the depreciation tax shield has become less significant. Clearly, at a higher discount rate and a higher inflation rate the right hand term would lose most of its significance as a contributor to the cash flow.

It is also apparent in the right hand term that the combination of taxes and inflation is the crucial factor. If there was a tax rate of zero the entire right hand term would fall out of the equation. If there is an inflation rate of zero the equation reduces back to equation [2.5].

To develop the internal rate of return analysis in the same way, start with equation [2.6] and modify it with the simplifying assumption that all cash outflows for investment \((C_j)\) occur at time 0. Then equation [2.6] becomes:

\[ [2.15] \quad - \frac{G_0}{(1+r)^0} + \sum_{j=0}^{n} \frac{(1-t)(R_j - E_j)}{(1+r)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+r)^j} = 0 \]
Now assume cash flows \((R_j - E_j)\) grow at a rate of \(f\) due to inflation, and that the denominators are also adjusted to real discount rate form:

\[
[2.16] \quad G_0 \frac{n}{(1+r)^0} \frac{(1-t)(R_j - E_j)(1+f)^j}{(1+r)^j(1+f)^j} + \frac{n}{(1+r)^j} = 0
\]

Cancelling and simplifying:

\[
[2.17] \quad G_0 + \sum_{j=0}^{n} \frac{(1-t)(R_j - E_j)}{(1+r)^j} + \sum_{j=0}^{\infty} \frac{tD_j}{(1+r)^j(1+f)^j} = 0
\]

Again, the same analysis applies as with \([1.14]\); a tax rate of zero will drop the right hand term; and inflation rate of zero will make the equation identical to \([1.6]\). The effect of the combination of taxes and inflation is the crucial point being made.

The model in this general form is adaptable to any depreciation method. Any cost-based method, including straight line, declining balance, and Accelerated Cost Recovery System will work in the equations. The particular method only has to specify the depreciation deduction in the given year \(j\).

Several generalizations can now be made about this model. In the statements 1-4 which follow, constancy is assumed for all variables other than those mentioned.

1. The tax rate \(t\) and the \(k\) or \(r\) will have an inverse relationship. A lower tax rate will imply a higher rate of return, with all other factors held constant.
2. The tax rate \( t \) and the net present value will have an inverse relationship, with \( k \) held constant.

3. The inflation rate and the discount rate will have an inverse relationship, with \( k \) held constant. Higher inflation will mean lower rates of discount will be necessary to produce the same amount of net present value at the point where the yield equation sum of inflows equals the sum of outflows.

4. The inflation rate and the net present value will have an inverse relationship.

Therefore, the cost of capital will be increased by the combination of inflation and taxes and should, therefore, create a depressing effect on potential investment.

When inflation was introduced into the analysis before [2.14] was developed, there was a simplifying assumption that all \( G_j \) took place at time 0. Now that assumption will be removed and the assumption will be instead that the \( G_j \) took place at time \( j \). In other words, multiperiod \( G_j \) will now be put back into the model.

However, before continuing with development of the model, it is worthwhile at this point to note that in the interpretation of the \( G_j \) term, the investment decision and the financing decision should not be confused with each other. For example, purchase of equipment on an installment contract represents both a financing decision and an investment decision. The fact that the equipment will be paid for over more than one period does not mean that there are multiple outlays, i.e., \( G_j \) in the capital budgeting equation. The equipment could have
been paid for with cash that was all equity financed. The decision to
purchase the equipment was the investment decision, and the decision
to use installment financing was the financing decision.

As a contrast, consider a project which has cash outlays over
many years, for example, development of a mine. The various cash
outlays would be relative to the investment decision in the manner of
the $G_j$ in the model which has been developed here. However, optimal
financing of those $G_j$ outlays is a separate financing decision, apart
from the decision to invest. Therefore, in putting the multiperiod $G_j$
back to the model, the $G_j$ are stipulated to be cash flows which relate
to the investment decision. They will not necessarily relate to the
financing decision.

The net present value form of the model will now be used to
elaborate the model for multiperiod $G_j$. If the $G_j$ were subject to
inflation, then they would inflate at $(1+f)^j$ but the discount rate
would, in turn, have to reflect the nominal discount rate $(1+k)^j(1+f)^j$
which would cause cancelling as in the following sequence:

$$[2.18] \quad V = - \sum_{j=0}^{n} \frac{G_j(1+f)^j}{(1+k)^j(1+f)^j} + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)(1+f)^j}{(1+k)^j(1+f)^j} + \sum_{j=0}^{tD_j} (1+k)^j(1+f)^j$$

Then simplifying and cancelling:

$$[2.19] \quad V = - \sum_{j=0}^{n} \frac{G_j}{(1+k)^j} + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)}{(1+k)^j} + \sum_{j=0}^{tD_j} (1+k)^j(1+f)^j$$
Thus, the $G_j$ would have a real cost behavior pattern similar to the $R_j$ and $E_j$. They would increase with inflation, and in discounting the inflated numbers, the same cancelling effect would occur.

As for the depreciation, it is conceivable that inflated $G_j$ would give rise to larger cost-based depreciation, but that depreciation would still get deducted in some subsequent period, so the effect which has been identified for depreciation would continue in any case.

An additional comment in passing is that the cancelling effect for $G_j$, $R_j$, and $E_j$ may not be generalizable to all cases in practice. It is conceivable that different costs, expenses, and revenues could grow at different inflation rates. In that case, it would be quite easy to use the model in the form stated at equation [2.18] and rename the numerator inflation rates as $(1+f_g)$, $(1+f_r)$ and $(1+f_e)$ for the outlays, revenues and cash expenses, respectively. For the denominator, the $(1+f)$ would become a $(1+f^*)$ reflecting the general price level.

Also, if it is desirable make certain inflation and uncertain inflation both explicit in the model, the denominator should be adjusted accordingly. Cooley, Roenfeldt, and Chew (1975) have advocated an additional discount term in the denominator to reflect the uncertain inflation. Thus, the nominal rate would represent a composite of the three terms, instead of the two discount terms which are explicit in [2.19] above.

The purpose of the model as it has been stated is for application in capital budgeting. Therefore, it is a normative model, intended to be used in a context of expectations.
The model has implications for a project, or a firm, or an industry. A firm is a bundle of projects, and an industry is merely a broader bundle of capital budgeting projects. The relationship that has been identified will imply increased capital costs at any of these levels.

The study will now turn to a review of the literature, including some empirical work that has been done on the concepts which have been illustrated in this model.
CHAPTER 3

REVIEW OF RELATED LITERATURE ON CAPITAL BUDGETING AND INFLATION

Literature on the topic of inflation is profuse. Even when the topic is restricted to capital investment vis-a-vis inflation, there are still many relevant sources of information. There are five groups that are relevant:

1. Historical literature regarding inflation. The various effects of inflation have usually depended on the magnitude of the inflation problem, varying from slow inflation to intermediate inflation to hyperinflation.

2. Theoretical models which describe the capital budgeting problem when inflation is present.

3. Empirical studies of the wealth-transfer effects which have been caused by inflation. Capital budgeting is integrally involved, as the return on investment, due to the effects of inflation, is affected.

4. Studies of security market returns in the presence of inflation. The securities markets are sources of investment capital, and thus are interrelated with capital budgeting.

5. Studies which focus on financial accounting, with an eye toward restating published accounting data, so as to emphasize the impact of inflation on the returns. For example, "ABC Corporation had 43% less income using replacement cost
The crux of the discussion will be the possible correlation of aggregate economic data for inflation and investment. Certain periods in several countries will be reviewed from library research. The material to be presented here is not intended to be any type of comprehensive survey, but seeks only to examine some significant trends at certain times in recent history. Indeed, in many references the search for data on both investment and inflation turned up little or nothing. Chou (1963, p. 105) in a book on the Chinese inflation of 1937-1949 said, "To estimate the investment for ownership benefit by the Chinese public during the inflation is a difficult undertaking because of the lack of comprehensive statistics." This statement was equally applicable to several other reference materials which initially
appeared to be promising sources of data, but contained very little useful data.

The magnitude of inflation has been categorized by Pazos (1972, pp. 13-19) as being "slow inflation, intermediate inflation, or hyperinflation." These categories are significant for the relationship being studied here regarding investment vis-a-vis inflation and taxes because the data available seems to show that the hypothesized relationship exists when intermediate or slow inflation occurs, but definitely does not occur in hyperinflation.

Slow inflation is the type of inflation experienced by the United States, Britain, France, and West Germany during the years 1949-1970 when most annual inflation rates were under 5% annual rates. The only exceptions when rates were over 6% for those nations in those years are 1949-1952 and 1958 in France when the rates were between 11% and 16%, 1951 in the United States and Western Germany when the rate was 8%, and 1951-1952 and 1970 in Britain when the rates were 8-9%. Pazos added that in countries with this type of inflation there is little or no tendency for the inflation to accelerate to higher and higher rates as in hyperinflation.

Countries characterized by intermediate inflation include Argentina, Brazil, Chile, and Uruguay where annual inflation rates in 1949-1970 ranged mostly between 10% and 40% with a few annual rates between 40% and 80%. Exceptions which are not in the 10%-80% range are 1949-1956 in Uruguay when inflation averaged 6.8% for those 8 years and 1968 in Uruguay and 1959 in Chile when inflation rates were 125% and 114%, respectively (Pazos, 1972, pp. 13-19). Intermediate
inflation does show some signs of spontaneous acceleration, but that
tendency is not really inherent in intermediate inflation (Pazos,
1972, p. 16). The fact that the tendency to accelerate is not inherent
may be due to government intervention. Pazos (1972, p. 84) reported
that when Latin American inflation rates exceed 40% annually "the
interval of wage contracts is shortened and inflation accelerates.
But the inflation frightens authorities and impels them to apply stiff
brakes, thereby bringing inflation back to its normal range."

Hyperinflation is the type of inflation where rates are over 80% annually, and continue to accelerate at an exponential rate. The pattern of acceleration can be easily seen in Table 1 taken from Pazos (1972, p. 18). Note that the percentages are given in monthly percentages. The magnitude of the hyperinflation in that table becomes even more dramatic when it is noted that an 80% annual rate is equivalent to a monthly compounded rate of 5%.

The significance of distinguishing hyperinflation from the other two types of inflation is that hyperinflation is an entirely different mechanism, and it creates an entirely different economic climate. In the slow or intermediate inflations the inflation rate is slow enough for necessary compensations to be made in the interest rate and other variables. In either slow or intermediate inflation the financial community can adjust adequately to the inflation rate changes which occur (Pazos, 1972, p. 17). However, in hyperinflation uncertainty and the speed of the inflation makes the interest rate a meaningless concept since monetary assets shrink daily by a significant amount. Therefore, the role of the rate of return on
Table 1. Hyperinflation--five selected cases (monthly percentage increases of internal prices during the last twenty months of hyperinflation and the first five after its termination)

<table>
<thead>
<tr>
<th>Sequence of Months</th>
<th>Germany a</th>
<th>Austria b</th>
<th>Hungary c</th>
<th>Poland d</th>
<th>Greece e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
<td>15</td>
<td>35</td>
<td>11</td>
<td>33</td>
</tr>
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<td>2</td>
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<td>7</td>
<td>23</td>
<td>16</td>
<td>6</td>
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<tr>
<td>3</td>
<td>9</td>
<td>4</td>
<td>24</td>
<td>34</td>
<td>11</td>
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<td>4</td>
<td>43</td>
<td>1</td>
<td>24</td>
<td>12</td>
<td>34</td>
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<td>5</td>
<td>91</td>
<td>15</td>
<td>-1</td>
<td>32</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>49</td>
<td>-10</td>
<td>2</td>
<td>37</td>
<td>36</td>
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<tr>
<td>7</td>
<td>97</td>
<td>32</td>
<td>15</td>
<td>26</td>
<td>47</td>
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<tr>
<td>8</td>
<td>104</td>
<td>33</td>
<td>9</td>
<td>58</td>
<td>90</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
<td>64</td>
<td>58</td>
<td>58</td>
<td>35</td>
</tr>
<tr>
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<td>43</td>
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<td>26</td>
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</tr>
<tr>
<td>11</td>
<td>145</td>
<td>46</td>
<td>13</td>
<td>7</td>
<td>152</td>
</tr>
<tr>
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<td>90</td>
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<td>14</td>
<td>43</td>
<td>3</td>
<td>62</td>
<td>63</td>
<td>158</td>
</tr>
<tr>
<td>15</td>
<td>139</td>
<td>16</td>
<td>20</td>
<td>72</td>
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<td>16</td>
<td>205</td>
<td>41</td>
<td>6</td>
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<td>1,276</td>
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<td>18</td>
<td>4,126</td>
<td>92</td>
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<td>148</td>
<td>1,909</td>
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<tr>
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<td>3,773</td>
<td>134</td>
<td>29</td>
<td>109</td>
<td>8,894</td>
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<tr>
<td>20</td>
<td>35,875</td>
<td>82</td>
<td>79</td>
<td>70</td>
<td>85,507,000</td>
</tr>
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<td>-10</td>
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<td>13</td>
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<td>63</td>
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<tr>
<td>25</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

a. April 1922-April 1924
b. February 1921-February 1923
c. July 1922-July 1924
d. June 1922-June 1924
e. April 1943-April 1945

investment as a device for efficiently allocating capital is severely disturbed.

Pazos (1972, p. 94) added that an essential feature which differentiates intermediate inflation from both slow inflation and hyperinflation is the continual pressure on prices from year-to-year caused by annual adjustment of wage contracts and other long-term commitments. This statement has interesting analogies to the United States and United Kingdom in the 1970s with the fact that automatic cost-of-living pay increases are now the rule in union negotiated wage settlements.

Slow Inflation--United States and United Kingdom

Empirical research which is relevant to whether or not inflation acts as a brake on investment in slow inflation situations will now be surveyed.

Corcoran (1977) presented a time series analysis of post-World War II capital formation. He compared the cost of equity capital with a weighted average figure which includes debt capital costs. In the 1970s the divergence between the two measures has varied from 1% to 5%, with the equity cost of capital below the weighted average cost. However, Corcoran pointed out that earnings/price ratios which use historical cost earnings are not comparable to earnings/price ratios of earlier years because of distortions in reported earnings which have been caused by inflation.

Corcoran (1977) also stressed the fact that the real relevance of the cost of capital measure is the effect that it has on the spread
between "returns on corporate assets and the cost of capital."

According to Corcoran the breadth of the spread provides the incentive to invest. The return on corporate assets involves reported earnings plus interest on debt in ratio to historical cost assets. The cost of capital involves a weighted average cost of capital (reported earnings plus interest) in ratio to market prices. The interesting finding is that there is actually a negative spread (i.e., cost of capital greater than return on corporate assets) for the years 1948-1958, 1970, and 1974-1976. The spread is positive in the 1959-1969 and 1971-1973. Corcoran pointed out that a measurement problem involved is that various inefficient and obsolete assets are mixed in the capital stock along with the newer ones. Also, some tax law changes may have contributed to this measurement problem. Students of finance would also criticize Corcoran's basis for this comparison, because the historical cost basis for return on investment which Corcoran used is a past results-oriented measure, while an earnings-price ratio is a forward looking measure. Nevertheless, for what it is worth, it is interesting that the "spread" has fluctuated from negative to positive to negative and in the face of higher inflation rates, it has been negative.¹

¹In comparison, for investors, real return on bonds remained negative for some extended period during the inflation of the 1970s in the United States. In the late 1970s and early 1980s, however, the real return on bonds not only became positive, but also reached a level substantially higher than had been true historically. The real rate of return on equity has exhibited a somewhat similar pattern, but the current 1982 positive real returns on stocks are not as high as the returns on bonds.
Kuznets (1961) has cited some long-term data which shows that there is a downward trend in gross capital formation as a proportion of gross national product when measured in constant dollar terms. Kuznets' data is compiled over the 1869-1955 period. He found that in constant dollars gross capital formation was 22.6% of gross national product in 1869-1888, 21.5% in 1909-1928, and 17.6% in 1946-1955. In current prices there is either stability or a small rise with 20.2% in 1869-1888, 20.9% in 1909-1928, and 21.3% in 1946-1955.

When net capital formation is expressed as a share of net national product, Kuznets found the share declined from 14.6% in 1869-1888 to 11.2% in 1909-1928 to 7.0% in 1946-1955. Kuznets (1961, pp. 396-398) also reported that the proportion of gross national product going to saving declined over the 1869-1955 period. In other words, investment in relative terms appears to have experienced a long secular decline.

Tatom and Turley (1978) pointed out that since the beginning of the economic expansion which began in early 1975, investment expenditures have grown at a 9.4% annual rate compared to an 11.8% average rate for the previous four recoveries. They converted their data to constant dollars and find that a 9.4% is 3.7% in real terms and the 11.8% rate is 7.8% in real terms. Therefore, the 1975 expansion investment growth rate is really only about one-half the growth rate of the previous four expansions when measured in real terms. Thus, with higher inflation, aggregate investment was less.

Table 2. provides an interesting comparison for the United Kingdom. Data on gross and net fixed domestic capital formation in
### Table 2. Investment in United Kingdom compared with inflation rates 1947-1956

<table>
<thead>
<tr>
<th>Year</th>
<th>Gross (Millions £, 1948 Prices)</th>
<th>Depreciation</th>
<th>Total</th>
<th>Annual Inflation Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947</td>
<td>1304</td>
<td>*</td>
<td>*</td>
<td>7</td>
</tr>
<tr>
<td>1948</td>
<td>1430</td>
<td>889</td>
<td>541</td>
<td>8</td>
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<tr>
<td>1949</td>
<td>1563</td>
<td>921</td>
<td>642</td>
<td>2</td>
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<tr>
<td>1950</td>
<td>1641</td>
<td>958</td>
<td>683</td>
<td>3</td>
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<tr>
<td>1951</td>
<td>1647</td>
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</tr>
<tr>
<td>1952</td>
<td>1671</td>
<td>1023</td>
<td>648</td>
<td>9</td>
</tr>
<tr>
<td>1953</td>
<td>1849</td>
<td>1062</td>
<td>787</td>
<td>6</td>
</tr>
<tr>
<td>1954</td>
<td>1995</td>
<td>1107</td>
<td>888</td>
<td>2</td>
</tr>
<tr>
<td>1955</td>
<td>2124</td>
<td>1157</td>
<td>967</td>
<td>3</td>
</tr>
<tr>
<td>1956</td>
<td>2234</td>
<td>1179</td>
<td>1055</td>
<td>3</td>
</tr>
</tbody>
</table>

*1947 depreciation unavailable in sources

Sources: Paish (1966, pp. 89, 140) and Pazos (1972, p. 17).
1948 constant pounds is given for the years 1947-1956. When these values are paired with inflation rates for the same years, it is obvious that gross investment grew by approximately 100 million pounds sterling per year, except 1951-1952 when the inflation rate was inordinately high at 9%. In those years investment growth was practically nil.

Intermediate Inflation: Latin America

Students of intermediate inflation, which occurs principally in Latin America, have suggested various theories which offer explanations of the cause of the inflation. These theories can be put into two groups: monetarist and structuralist. In the monetarist view, erroneous policies such as price controls, foreign trade policy, fiscal policy, and monetary policy have caused inflation and stagnation. On the other hand the structuralist points out such inefficiencies as stagnation of food supply in the face of expanding demand, inelasticity and instability of the purchasing power of exports, bottlenecks in the supply of capital and skilled labor, and structural deficiency in the tax system (Lioi, 1974, pp. 3-7). One of these views may be held by the economic decisionmakers in a particular country, and policy may reflect one viewpoint to the exclusion of the other (Lioi, 1974, pp. 3-7; Kirkpatrick and Nixson, 1976, pp. 130-133, and Wachter, 1976, Chapter 1).

The choice of either a structuralist or monetarist orientation will have particular implications for the investment climate. For example, Brazil apparently uses an "inflation tax" which is really a
product of money creation and a nominally constant tax structure with progressive rates. The theoretical effect is to give the government a proportionately larger share of the national product each year, which in turn is to be spent on autonomous investment by the government. However, whether or not this actually occurs is dependent on the cost elasticity of the demand for money (Pastore, Almonacid, and Barros, 1977, pp. 15-16; Lemgruber, 1977, pp. 69-71).

Advocates of the "inflation tax" to stimulate the economy point out that government spending on investments is autonomous while private investment must be induced. Creeping inflation is supposed to be a stimulus which releases additional purchasing power in anticipation of increases in prices; in so doing some of the purchasing power is hopefully diverted into investment (The Indian Merchants' Chamber Economic Research and Training Foundation, 1961, p. 61). The central bank and the rest of the banking system will share in the proceeds of the inflation and are counted on to redistribute it back into the system with credit expansion (Lioi, 1974, p. 110; Cagan 1956, pp. 77-83).

Samant (1975, p. 24) has pointed out that the induced private investment depends on the incentive provided by the difference between the rate of interest on borrowed funds and the marginal efficiency of capital. If an increase in money supply by the government fails to decrease the borrowing rate, there will be no direct stimulus in the private sector. However, government investment could provide indirect stimulus.

The effectiveness of the "inflation tax" has been evaluated by Kahil (1973, p. 157) who reported that since almost all of the
direct taxes in Brazil and 60% to 70% of the indirect taxes are borne by the upper income groups, the inflation tax is coming to a large extent out of potential saving. However, the official rationale is that if the government uses all of the "inflation tax" for investment, there will be a net increase in investment. However, Kahil pointed out that instead of the Brazilian government increasing investment, consumption was increased. In fact, consumption was increased to such an extent during the 1947-1963 period that savings fell, and Gross Domestic Product for many of the years involved was actually below the 1947-1950 level.

Kahil (1973, p. 182) was also very critical of the choice made by the government for investment projects. His case in point is the assertion that construction of Brasilia in the wilderness of Goias and building highways connecting the new capital to the rest of the country meant fewer schools, houses, roads, and less electricity, transport and other services in population centers that could have used them.

Kahil's (1973) extensive study of Brazilian inflation led him to the conclusion that inflation was of doubtful value as a means of channeling investment into more desirable fields or of accelerating investment. He found that there was not any evidence of redistribution from consumption to savings sectors of the population. Kahil concluded that the inflation tax transferred a significant proportion of resources from the more efficient private sector, and in effect, wasted a precious resource, capital (Kahil, 1973, p. 328).

Kahil has also cited evidence that inflation actually depressed investment. This is presented in Table 3. Kahil said that the fact
Table 3. Brazil: Comparison of inflation and percentages of Gross Domestic Product (GDP) for investment and saving

<table>
<thead>
<tr>
<th>Year</th>
<th>Yearly Percentage Change in Prices (GDP Implicit Deflator)</th>
<th>Gross Private Investment as Percentage of GDP (1953 Prices)</th>
<th>Gross Domestic Private Saving as Percentage of GDP (1953 Prices)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947</td>
<td>13*</td>
<td>14.2</td>
<td>11.4</td>
</tr>
<tr>
<td>1948</td>
<td>10</td>
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<td>11</td>
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<td>14.9</td>
<td>10.6</td>
</tr>
<tr>
<td>1952</td>
<td>13</td>
<td>15.0</td>
<td>12.2</td>
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<td>1953</td>
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<td>12.0</td>
<td>13.2</td>
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<td>1960</td>
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<td>16.5</td>
</tr>
<tr>
<td>1963</td>
<td>78</td>
<td>10.0</td>
<td>12.4</td>
</tr>
</tbody>
</table>

*1947 Price change is old Conjuntura Económica Index

Source: Kahliil (1973, pp. 130, 338).
that Gross Private Investment as a Percentage of Gross Domestic Product dropped over the years 1949-1963 while inflation increased is proof that inflation and investment moved in opposite directions. Kahil (1973, pp. 130-131) added that his table would have been even more dramatic if he had included data for coffee plantings in 1947-1950 which were about 2%-3% of Gross Domestic Product followed by 1%-1.5% of Gross Domestic Product for the years 1951-1954 after which time they fell to zero.

In a study of inflation in Argentina, Brazil, Chile, and Uruguay, Pazos (1972, pp. 41-42) tabulated "Annual Investment in Fixed Assets and Changes in Inventories" as a percentage of Gross Domestic Product to see if there is a correlation with inflation rates. He concluded that the two series do not correlate closely and do not support any idea of relation between them. He further concluded that the comparisons do not support either the idea that inflation facilitates investment by forcing up saving, or the idea that it keeps investment down by discouraging saving.

Beside his attempt to correlate investment and inflation, Pazos also considered the Fisher MV = PT identity components to see how well they correlate with inflation. Pazos (1972, p. 110) said he found as many combinations of the variables as are possible, while still preserving the identity. $R^2$ is tabulated in Table 4 for money supply correlated with prices, and for velocity correlated with prices for 1949-1970 data.

One surprising finding was a negative association between monetary velocity and real growth, which is contrary to the case of
slow inflation economies. For the developed economies an increase in velocity is usually associated with economic expansion and a decrease in velocity with a depression (Pazos, 1972, pp. 115-117).

The literature that has been briefly cited here provide a sampling of the complexity of interrelated factors and the fact that perfect correlations of theoretical relationships are often not found in aggregate economic data.

Hyperinflation: Germany 1920-1923

In Germany in 1920-1923 hyperinflation encouraged excessive investment. The process was literally a flight from cash. In the later stages of the inflation money depreciated hourly and had to be gotten rid of as fast as possible. The German Wholesale Price Index (January, 1913 = 1.0) was at 14.4 in July, 1921, and at 750,000,000,000.0 on November 15, 1923 (Ringer, 1969, p. 80). Anyon
with monetary assets felt that they had to turn the money into real value as quickly as possible. This led to excessive investment in plant and equipment by business. Much of this investment is generally conceded to have been inefficient (F. D. Graham, 1930, p. 323; Bresciani-Turroni, 1937, p. 381; Laursen and Pedersen, 1964, p. 95).

Even so, many businessmen made huge profits from investments made with borrowed money. The procedure was to obtain short-term bank credit for working capital and investment in plant. The money was then spent at frantic speed, limited only by the availability and speed of acquisitioning of materials and manpower. When the short-term debt came due, payment was made with devalued money; plant and equipment had thus been obtained at very little cost. The demand for this type of financing grew; interest rates soared but still lagged behind the rate at which the currency was depreciating. The inflation became so intense that the currency printers could not keep up with the demand for paper money (Ringer, 1969, pp. 81-82).

The German short-term loan rate rose as high as 20% per day, or 7,300% per year (Robinson, 1938, p. 511). The Reichbank (Central bank) was reluctant to raise interest rates to market levels. F. D. Graham (1930, pp. 65-66) said that the Reichbank was practically giving money away at a 900% annual rate in September, 1923. With the chaos which existed in interest rates, corporate bonds appeared which required payment in real commodities including anthracite, electricity, potash, and cement (Schacht, 1967, p. 65).

Much of the capital which produced this frenzied atmosphere was later scrapped in the period of "rationalization" which followed.
In many instances, the capital created in the hyperinflation had no profit and loss analysis made to justify it. Little effort was made to distinguish between investments that would have a positive future yield and investments that would prevent erosion of the money value which they represented (F. D. Graham, 1930, p. 323). However, even though a good many of these investments proved useless, there apparently was a marked betterment in industrial equipment on the whole (F. D. Graham, 1930, pp. 242-243).

One symptom, or perhaps a result depending on the point of view, of hyperinflation is a tendency for the aggregate value of the money in circulation to decrease. In effect, prices increase faster than money supply does, even though the printing presses are running at full tilt to supply the money. Table 5 illustrates the 1913-1922

Table 5. Decrease in real value of money in circulation in Germany, 1913-1922

<table>
<thead>
<tr>
<th>Date</th>
<th>Real Value of Circulation (Millions of Marks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1913 Average</td>
<td>6,070</td>
</tr>
<tr>
<td>1921 September</td>
<td>4,560</td>
</tr>
<tr>
<td>1921 November</td>
<td>3,175</td>
</tr>
<tr>
<td>1922 July</td>
<td>2,560</td>
</tr>
<tr>
<td>1922 December</td>
<td>880</td>
</tr>
</tbody>
</table>

Source: Bresciani-Turroni (1937, p. 162).
German example of a decrease in total real value of the circulation, computed with the wholesale price index and the nominal money supply. Obviously the velocity had to have increased to permit an almost five-fold drop in the real value over the 1921-1922 period.

In Germany, in 1923 the value slipped further and oscillated among various levels. Bresciani-Turroni (1937, pp. 160-164) has provided data showing similar trends in hyperinflations in Austria, Russia, and Poland. The tendency for prices to rise faster than the money supply seems to be slight or nonexistent in intermediate inflation situations. Bresciani-Turroni (1937, pp. 165) also compared Italy, France, Holland, and Switzerland during the 1913-1921 period when those countries experienced intermediate inflation and the prices did not rise as fast as the money supply.

F. D. Graham (1930, p. 107) observed that in Germany in August, September, and November of 1923 prices did not rise as fast as the rate of increase of the money in circulation. He concluded that in those 3 months the ceiling in the rate of monetary turnover had been reached.

It seems paradoxical that excessive investment occurred simultaneously with a shrinkage in the total value of circulating money. However, production of consumption goods and goods for export fell drastically. Therefore, there was productive capacity available for investment to increase as it did (F. D. Graham, 1930, pp. 302-305).

Obviously, even though hyperinflation is very interesting, it does not seem to be applicable as a historical situation to examine regarding the interaction of income tax and inflation which
was theoretically modeled in Chapter 2. The primary investment motivation in Germany in 1920-1923 was to turn money into some real asset form, and the return on the investment in money terms was secondary. Indeed real return on investment was the motivation to invest as quickly as possible. Yet, the form of the potential return was a race for ownership of more real assets. Returns earned from business operating profits seemed to be subordinate to the desire to get out of money and into more real property. Since the rate of return is really the crucial profitability criterion in the theory in Chapter 2, hyperinflation can be dismissed as interesting, but probably not a very good source of information to compare to economies which are experiencing slow inflation.

On the other hand, intermediate inflation and slow inflation seem to generally confirm the theory in Chapter 2 from a historical perspective. However, there are multiplicity of factors present in aggregate economic data which make simple correlations somewhat suspect. Therefore, in the next two groups of literature, theoretical studies and empirical studies, the relationship of inflation and investment will be expanded principally with United States data.

Theoretical Models on Inflation in Capital Budgeting

The second group of literature to be reviewed emphasizes theoretical models of the inflation-to-depreciation problem. A significant model has been presented by Nelson (1976b). Nelson started from the premise that in the economics literature, it has been shown that price inflation in a world of income taxes depresses capital
investment. He expressed the premise explicitly as follows: Assume that a firm invests a sum I which produces a cash flow of X one period later. At that time the project would be fully depreciated, the taxable profit would be X-I, and the tax would be t(X-I). With discount rate r the present value of the project is:

\[ PV = -I + \frac{X - t(X-I)}{1 + r} \]

If inflation is anticipated at a rate of p over the life of the project, the present value is then

\[ PV(p) = -I + \frac{(1+p)X - t((1+p)X-I)}{(1+r)(1+p)} \]

\[ = -I + \frac{(1-t)X}{(1+r)} + \frac{tI}{(1+r)(1+p)} \]

The optimal level of investment is given by

\[ \frac{3PV(p)}{3I} = -1 + \frac{(1-t)3X}{(1+r)3I} + \frac{t}{(1+r)(1+p)} = 0 \]

Nelson then uses the approximation \((1+p)^{-1} \approx (1-p)\) to rearrange the above equation as

\[ \frac{3PV(p)}{3I} = -1 + \frac{(1-t)}{(1+r)} \frac{3X}{3I} + \frac{t}{(1+r)} - \frac{t}{(1+r)} \]

The term at the extreme right, \(\frac{dt}{(1+r)}\), influences the solution such that a higher rate of anticipated inflation will result in a downward shift in the marginal present value. The relationship is
graphed in Figure 1 for a situation where an inflation rate of $P_2$ is higher than an inflation rate of $P_1$. Thus optimal investment is smaller for the higher rate of inflation.

Nelson used related theory to develop five propositions:

1. **Proposition A.** The optimal level of capital investment will depend in general on the rate of inflation. The amount invested will typically be smaller the higher the rate of inflation.

2. **Proposition B.** The rate of inflation will influence the firm's choice of technologies of production through its choice of a capital/labor ratio. Higher rates of inflation will typically be associated with lower capital/labor ratios.

![Figure 1. Inverse relationship between marginal present value and anticipated inflation](image)
3. **Proposition C.** The net present value ranking of mutually exclusive investment projects will depend in general on the rate of inflation.

4. **Proposition D.** Net present value rankings of mutually exclusive projects which differ in respect to durability will depend on the rate of inflation. Typically, rankings will change in favor of projects with lower durability at higher rates of inflation.

5. **Proposition E.** Replacement policy will depend in general on the rate of inflation. The higher the rate of inflation the more likely will replacement be deferred to a future period.

The Nelson model has been extended by Settle (1978) into a comprehensive theoretical model of firm behavior. His results are produced by performing comparative statics on a Lagrangian optimization problem. Conclusions are that inflation expectations would adversely affect production, investment, and the value of the firm, but will motivate the firm to increase the debt in the capital structure. However, neither Nelson nor Settle offered any empirical studies of their results, other than some numerical examples of the formulas which they developed.

Several other theoretical studies develop the point that depreciation is worth less in present value terms when inflation occurs. These studies include articles by Bierman and Smidt (1975), Terborgh (1960), Hong (1975), Gandolfi (1976), Jaffe (1978), Raiborn and Ratcliffe (1979) and Cross (1980). All of these authors go through
a modeling procedure somewhat similar to that which was presented in Chapter 2 of this study, and they arrive at theoretical conclusions similar to those given in Chapter 2.

Also relevant are theoretical models which use versions of the capital asset pricing model to derive inflation related valuation equations. Such models have been presented by Chen and Boness (1975) and Roll (1973).

**Empirical Studies of Inflation-caused Wealth Transfers**

The third group of literature to be reviewed considers the wealth transfers caused by inflation. The framework of these studies typically is to use a series of balance sheets to determine whether or not the firm has benefited from inflation, or has been hurt by inflation, over a period of years. In these studies the inflation-sensitive balance sheet items are specifically analyzed. The three inflation-sensitive balance sheet items include the net monetary debtor-creditor status of the firm, the depreciation of the plant and equipment, and the inventory.

The debtor-creditor status of the firm is inflation-sensitive because to the extent that a firm is a net debtor, those debts will be paid off in inflated dollars. Depreciation is sensitive for the reasons which have been discussed in Chapter 2 of this study, and therefore, nominal earnings are overstated compared to real earnings, since depreciation is inadequate. In regard to inventory, the firms which use first-in, first-out or average cost inventory accounting are understating their cost of goods sold. In inflation, they will
typically face higher costs to restock inventory, as compared to the amounts being expended as cost of goods sold. Thus, they overstate profits and pay more taxes than they would if replacement cost, or even last-in, first-out, would have been used for inventory costing. Empirical studies on these points will now be discussed.

Hong (1975, 1977a, 1977b) was quite thorough in his studies. In Hong's dissertation (1975) he used all three factors plus systematic risk as independent variables in a multiple regression test. Hong used systematic risk as a variable because security market returns have been shown to have systematic linear relationship, and therefore Hong believed that his data would be biased if the systematic risk was not included as one of the regression variables. After determining the wealth transfer effects of the three inflation-sensitive factors, Hong then tested whether or not the stock market had effectively priced the wealth transfer information into share price. He concluded that it had done so and, therefore, wealth transfer effects had occurred in terms of share prices. Hong's work is based on 1950-1968 data.

Hong (1975, p. 4.27) concluded: "Firms with a high proportion of wealth in fixed assets suffer greater losses than those with low fixed assets." He (p. 6.3) added: "Such losses must seriously impair the ability of firms to make new investments from retained earnings. They also have obvious important bearing on the current so-called capital shortage problem in the United States."

In a similar study, Bryan (1975) did more elaborate model building than Hong in regard to the three inflation-sensitive factors,
but Bryan used a limited sample of security prices in two years where Hong used 19 years.

Bradford (1974) and Alchian and Kessel (1959) included depreciation in their work, in the same context as Hong (1975). These studies have the same conclusion as Hong; i.e., that wealth transfers are empirically provable. An important contribution by Hong was the inclusion of the systematic risk in the regression, and he achieved higher statistical significance in his results compared to the earlier studies.

Kim (1979) also found net operating income to be sensitive to inflation and the size of depreciation charges, due to the inflation-tax effect. However, Gonedes (1981) has refuted Nelson (1976b), Hong (1975, 1977a, 1977b) and Kim (1979) with empirical data that shows that liberalized depreciation allowances adequately compensate for the asserted inflation-tax effect.

A group of related studies consider the net monetary debtor-creditor status of the firm without also considering the depreciation or inventory issues. Net monetary debtor creditor status and systematic risk in security market returns are used as regression variables in a test of wealth transfer effects by Bach and Stephenson (1957). They also considered debtor-creditor status compared with a rate of return in a test of significance. This test was structured with debtor-creditor status used to group the firms in the sample, and then a test of significant differences was made on the returns of one group compared to the other. A significant difference was found.
Other studies which test the debtor-creditor status cause wealth transfer effects include articles by Kessel (1956), de Alessi (1963), Kessel and Alchian (1960), Bach and Ando (1957) and Broussalian (1961). All of these studies confirm that inflation caused wealth transfers from creditor to debtor, except for de Alessi who did not find that effect in certain British data.

Dietrich (1981) found similar wealth transfer effects in a study which focused primarily on bonds.

The Effects of Inflation on Security Market Returns

The fourth group of literature to be reviewed includes those studies which examine the extent to which stocks or bonds are inflation hedges. This group of literature is relevant because the securities markets are proxy measures for investors' attitudes toward new investment. Hence, security market returns relative to inflation represent attitudes toward investment in the presence of inflation.

The studies which have tested the extent to which securities are inflation hedges include articles by Lintner (1973, 1975), Bodie (1976), Nelson (1976a), Jaffe and Mandelker (1976), Reilly (1975), Reilly, Johnson, and Smith (1970), and Olsen (1980). In these studies inflation and returns on market value are the variables correlated. The typical conclusion is that stocks have not been an effective inflation hedge, and if any relationship did exist, it was that perhaps stocks should be sold short. Reilly (1975) pointed out that stocks were a poor hedge against unexpected inflation, when the inflation is divided into anticipated and unanticipated portions. Olsen (1980) has
found that holding period returns vary inversely with the expected rate of inflation, but directly with uncertainty about the expectation of inflation.

Empirical testing has been done by Derosa (1978) who generally confirmed a conclusion that the stock market does price the effects of inflation into stock price movements. However, he refuted Lintner's hypothesis that inflation dilutes shareholder interest in a firm.

Fama (1981) has suggested that the relationship between real stock market returns and inflation represents proxy effects. He concluded that a better comparison would be between real productivity and real stock market returns.

Financial Accounting Literature on Inflation Adjustments

The review of literature will now shift to financial accounting literature which discusses inflation. However, before elaborating on the literature in the fifth group, it should be pointed out that accounting for financial statements and accounting for tax purposes generally do not have to coincide. The two accounting bases need to coincide for only a few items, notably inventory methods, under the United States Internal Revenue Code. Depreciation is one of the areas for which different methods may be used for book and tax purposes.

Therefore, financial accounting and tax accounting are essentially two separate problems with regard to the depreciation-tax-inflation problem. Financial accounting has made significant progress in requiring the effects of inflation be disclosed in supplementary statements. This is the approach in Financial Accounting Standards
Board Statement No. 33 (1979). Statement No. 33 requires disclosure of supplementary information on both a constant dollar and a current cost basis by large publicly held companies. The Financial Accounting Standards Board has also issued Statements 39, 40, and 41 which deal with price-level problems in mining, oil and gas, timber, and real estate. Adjustments are made by use of general price-level indices.

A similar approach, i.e., requiring supplementary information in published financial statements, is also required or proposed by the respective accounting bodies in the United Kingdom, Australia, New Zealand, and Canada (Brennan, 1979; McGee, 1981).

Some of the literature which discusses the impact of inflation on accounting statements includes studies by the Financial Accounting Standards Board (1977), Hong (1977b), W. J. Graham (1959), Brown (1952), Edwards and Bell (1961), and Vatter (1962). These studies show the impact on reported earnings when inflation effects are included. For example, Hong (1977b) included a chart for 25 companies where he has estimated the percentage of net income which was lost to over-taxation due to under-depreciation in 1972.

Edwards and Bell (1961) have written the classic book on the topic of adjusting accounting statements for the effects of inflation. Vancil and Weil (1976) have included an annotated bibliography of inflation accounting literature. Jones (1956) has presented a thorough theoretical treatise on the deficiencies of historical cost based accounting in the face of inflation.

Two collections of articles which survey the status and issues on inflation accounting worldwide are by V. K. Zimmerman (1979)

Implementation of Current Cost Accounting, or Constant Dollar Accounting, are the themes of books by Miller (1980), Kirkman (1974), and Davidson, Stickney and Weil (1976).

Sandilands (1975) chaired the British Parliament's select committee report which thoroughly evaluates the alternative accounting methods available for inflation accounting. The book then makes recommendations for current value financial statements which would be the only statements, rather than the supplementary approach in FASB 33. This book has been characterized as a major contribution to the accounting literature by Vancil and Weil (1976, pp. 10-11) who added that "This 'Sandilands Committee Report' is the one book on accounting we would own if we could own only one."

The Inflation Accounting Steering Group of the British Chartered Accountants published a Guidance Manual on Current Cost Accounting in 1976 as an exposure draft of the British equivalent of FASB 33. However, practitioners balked at the mandatory current value accounting. A more flexible version, ED24, has been issued which takes an approach similar to the supplementary data in FASB 33 (Iqbal, 1981, pp. 153-165).

A book of articles on inflation accounting issues has been edited by Dean and Wells (1978), with emphasis on British and Australian proposals for inflation accounting in financial statements.
In regard to the separate problem of inflation in tax accounting, the United States tax laws still require cost-based depreciation despite rapid writeoffs and investment tax credits. Parker and Zieha (1976) have suggested a model to measure the extent to which accelerated depreciation and investment tax credits overcome the problem of historical cost based depreciation, as was illustrated in Chapter 2. They found those provisions to be inadequate, using 1975 data.

The theoretical answer to the problem is indexation of the tax law. In other words, let the taxpayer pay tax upon the real value of the profit. For example, studies advocating indexation are by Aaron (1976a, 1976b). Lent (1976) has written on the experiences of various foreign countries in trying to cope with inflation in tax law and policy. He reported that Canada, United Kingdom, the Scandinavian countries, Austria, Belgium, France, West Germany, Italy, Spain, Indonesia, Israel, Japan, and Korea have all taken steps at one time or another to deal with inflation, but generally these have been one-time revaluations of the books.

Lent added that Brazil and Chile have a form of indexing of taxes. Argentina has also used indexing (Miller, 1980, p. 71). Lent (1976, pp. 210-211) said that these countries which have permitted some type of inflation adjustments have generally experienced compliance and administration problems.

None of the European or other industrialized nations cited above have ever experimented with indexing on business profits. Brazil, Chile, and Argentina are apparently the only examples.
However, it should be noted in passing that there have been inroads made in the area of indexation of personal income taxes. Indexing is now operative in Canadian personal income taxes and the personal income tax laws of several states of the United States. The Economic Recovery Tax Act of 1981 contains indexing of personal tax brackets, personal exemptions, and the zero bracket amount, beginning in 1985.

A clarification of the concept of indexation should be added at this point. Indexation as discussed above focuses only on indexation of the tax law or accounting statements. Some economists have proposed a broader concept of indexation, involving complete indexation of everything which is fixed in nominal money terms. This would include bonds and other debts, contracts, and regulated prices. Elaboration of this concept can be found in studies by Sarnat (1976), Friedman (1974), Morag (1962), and Bach and Musgrave (1941). Some isolated examples of bond indexation are cited by Sandilands (1975, p. 234). Further discussion of comprehensive general indexing is beyond the scope of this study.
In this chapter the economics literature on capital resources exploitation will be reviewed. Then, in the subsequent chapter, the theory from Chapter 2, and models from the literature in this chapter will be combined to hypothesize that the capital budgeting problem involving taxes, inflation, and depreciation can also affect the rate of exploitation of a natural resource.

The literature to be reviewed here initially will focus exhaustible resources (minerals, etc.). Renewable resources (e.g., forests) will be discussed later.

The rate of resource exploitation has often been analyzed in the economic literature as a capital budgeting optimizing decision. Indeed, a natural resource stock, when owned by a firm, is a part of the productive plant and equipment of the firm, and is also the source of the cash flows to the firm. In this vein, the theory of resources exhaustion in economics holds that an optimal pattern can be determined for the removal of the resource, given demand and capitalization rate parameters. In effect, the theory considers whether or not it is profitable to limit current output so that production would be possible in future periods also. The resource really has a two-fold character; it has investment value, and it provides income. Thus, the connection to investment present value theory in finance is obvious.
Another context for analysis of natural resources exploitation is the consideration of exploitation rate vis-a-vis optimal utility for society, i.e., social welfare theory. Indeed, this is the focus of much of the economics literature. Some of the literature of that type will be noted later in passing, but the focus here will be on the exploitation rate as it relates to capital budgeting.

The first article on the topic of optimal resource exploitation rate is generally thought to be the work of Gray (1914). Gray presented the concept that maximization of returns was possible for a firm, provided demand parameters were known. Gray used numerical examples and verbal analysis to suggest that with a given capitalization (interest) rate and demand function, an exploitation schedule could be calculated to maximize the present value of returns. Gray is acknowledged to be the first writer to suggest that the present value of a mineral resource must reflect the interest rate during the period of exploitation.¹

Alfred Marshall (1920, pp. 438-439) referred to a "royalty" return for the owners of mines, but his discussion really only hints at the concept of an optimal pattern of removal of the resource in order to maximize the royalty. Also, the context of the brief reference by Marshall is really a discussion of rents and royalties as related to the findings of Adam Smith and David Ricardo. Yet, the

¹Gaffney (1967, passim) and R. L. Gordon (1967, p. 275) imply this conclusion. None of the literature surveyed cites any writer prior to Gray who discussed this point. Gray himself footnoted some writers who discussed rents and royalties, however apparently none of them considered the relationship of the interest rate and maximization of present values.
mention of mines was really an example which was stated in such a way that it could be concluded that Marshall was implying some things which were stated explicitly by others in later writings.

The most complete pioneering treatment of the issues involved was by Hotelling in 1931. Hotelling's contributions in that article are numerous and will be reviewed in some detail. The procedure will be to give an overview, followed by some specific references.

Hotelling gave the topic a rigorous mathematical discussion and concluded that the price of the unrecovered resources (in the ground) must rise at the rate of interest. Hotelling's analysis is based on a premise that in equilibrium the resource owner must be put in a position of being indifferent between producing today and producing tomorrow. According to Hotelling, the interest rate is the price tomorrow less than today's price plus interest, production today would be encouraged. On the other hand, if the price tomorrow exceeds today's price plus interest current production will be deferred.

The value of a resource deposit is the present value of the future production, less costs. The resource owners will be expecting the net price of the resource to be increasing exponentially at a rate equal to the rate of interest. In the microeconomic model of a competitive market, price is equal to cost plus a normal profit margin. In the Hotelling analysis, price reduces to cost plus a profit margin which reflects the cumulative compound interest factor to the particular point in time.
According to R. L. Gordon (1967, p. 274) a primary contribution of Hotelling is a more precise definition of exhaustion. Hotelling (1931, p. 141) defined the time of final exhaustion as time $T$ and the exploitation period is then a period from time $= 0$ to time $= T$, and the quantity exploited over that period will be $q$ such that

$$
\int_0^T q dt = \int_0^T f(\text{po} e^{-\gamma t}, t) dt = a
$$

where $\text{po} e^{-\gamma t}$ is the price at time $= 0$ expanded as an "e" continuous compounding function of time $= t$ at interest rate $\gamma$, and $a$ is the total supply. Quantity is $q = f(p, t)$.

With an assumed demand function of $q = 5- p$ provided $q = 0$ for $p \geq 5$ where $p$ is price and $q$ is quantity, Hotelling proceeded to algebraically manipulate so as to show that there is only one point where the price (and demand) function is tangent or intersects the supply function, and this point is the time of exhaustion of the resource.

Hotelling (1931, p. 142) elaborated that the nature of the demand curve will govern whether or not actual exhaustion will indeed occur, or production will diminish gradually approaching zero at infinity. If the demand curve is of the form $q = A + Bp$ where $q$ and $p$ are quantity and price and $A$ and $B$ are constants, there will be a finite time of exhaustion. If the demand function is of the form $q = e^{-bp}$ then the exploitation will continue forever as a diminishing function (where $b$ is a constant).
Hotelling (1931, pp. 150-152) also considered social value and utility theory. He pointed out that because a monopolist equates marginal cost with a downward sloping marginal revenue curve, there is a reduction in output so as to earn maximum profits. This is the typical microeconomic theory monopoly profit exercise. However, Hotelling added to the typical monopoly model his conclusion that the monopolist will also be subject to the requirement that the price of the resource must continue to rise proportionately to reflect the cumulative interest on each unit produced.

Hotelling considered the effects of taxes on the solution. He concluded that a capital value tax has no effect other than to "transfer to the government treasury a part of the mine owner's income." This statement reflects the fact that Hotelling regarded taxes on the capital value of a mine to include an income tax, since present value of the mine was really just income receivable over time in the Hotelling definition of value.

His proof proceeded as follows. The assumed tax rate is a, levied on value J at time t, so the value at time τ is:

\[ J(\tau) = \int_{\tau}^{T} [pq - aJ(t)]e^{-\gamma(t-\tau)}dt \]

which reduces to a differential equation as

\[ J'(\tau) = -pq + aJ(\tau) + \gamma J(\tau) \]

"The constant of integration is evaluated by means of the condition that \( J(T) = 0 \)." (Hotelling, 1931, p. 164). Then
\[ J(\tau) = \int_{\tau}^{T} p q e^{-a(t-\tau)} dt \]

which shows a to be added to \( \gamma \), and therefore a is only an increase in the capitalization rate.

Hotelling's analysis of a severance tax concluded that conservation is encouraged by the severance tax. His definition of a severance tax specifies that it is a fixed number of cents (or dollars) per unit of output. The solution that conservation will result seems intuitive since a severance tax defined as a fixed amount per unit means that variable costs increase by a constant, so that demand and supply curves will have to intersect at a higher price. We note this in passing, because a severance tax defined as a constant percentage of the market value of output will give a different solution. This alternative definition of the severance tax will be considered later in connection with another author, Herfindahl (1967).

Another contribution made by Hotelling in his pioneering article is analysis of the concept that cumulated production will affect price (pp. 152-157). Simply stated, exhaustion costs increase as a mine goes deeper, plus the fact that the scrap market accumulation can also influence the market price. Hotelling's analysis here is done by several special cases to which the calculus of variations is applied.

Gaffney (1967) has edited a symposium of articles by various authors on the topic of extractive resources and taxation. In this collection, Herfindahl (1967, pp. 68-90) suggested a graphic representation of the process by which the interest rate affects resource
prices. Figure 2 reproduces Herfindahl's graphs. In the left panel the slope of line a reflects the interest increment. Line c assumes constant extraction cost through time. A particular point in time will dictate the price that must be charged to cover extraction cost and cumulative interest. The price so determined in turn is found on the vertical axis of the demand schedule (the right-hand panel). A line down to the horizontal axis represents the quantity of the resource to be produced at that point in time.

Herfindahl also suggested another graph which illustrates the reaction the Hotelling model would predict for cost changes or interest rate changes. That graph is reproduced here as Figure 3. In short, the area under the curve represents resource to be extracted, and the vertical lines to points \( T_0 \) and \( T_1 \) at the right represent potential times of exhaustion. A reduction in interest rate is represented by the dotted line and a longer extraction period results.

Figure 4 depicts a cost decrease moving \( C_0 \) to \( C_1 \) in which the period of extraction shortens. Conversely, a cost increase could be represented by movement from \( C_1 \) to \( C_0 \) and the period of extraction lengthens. In Figure 5 an increase in demand shifts up the entire curve, and the period of extraction is shortened.

Herfindahl (1967, p. 70) pointed out that a severance tax of a constant percentage will have no effect on output prices, production, or consumption. The reasoning assumes that when the tax takes a percentage there is no advantage derived from deferring production, as there was in the Hotelling constant dollars and cents per unit severance tax. Basically, a percentage severance tax is the same
Figure 2. Relationships of price and quantity over time for metal being mined

Figure 3. Effect upon production period caused by reduction in discount rate

Source: Herfindahl (1967, pp. 63-90). Copyright, University of Wisconsin Board of Regents—used by permission.
Figure 4. Effect upon production period caused by decrease in cost

Figure 5. Effect upon production period caused by upward shift of demand curve

Source: Herfindahl (1967, pp. 63-90). Copyright, University of Wisconsin Board of Regents—used by permission.
concept which Hotelling examined under the name "capital value tax" with the same conclusion.

Herfindahl also had numerous other publications on natural resource topics which are collected in Resource Economics: Selected Works of Orris C. Herfindahl, edited by David B. Brooks (1974). In that collection there are selections on conservation, market structure of the minerals industries, and exploration strategies. Also Herfindahl (1959) presents a portfolio theory for investment in mineral acreage in different areas. A discussion of risk and return is included.

Herfindahl and Kneese (1974) co-authored Economic Theory of Natural Resources which has several sections on capital investment. This book is textbook length and the topics are varied, including cost and production theory from microeconomics, dynamics capital theory, cost-benefit analysis relating to government investment, and "residuals from natural resources," i.e., externalities. The sections of the book which deal with investment describe discounted cash flow analysis techniques such as net present value, etc. in the style of a finance text.

The Gaffney (1967) collection contains several other significant articles with relevancy to this study. Articles by Seagraves, McDonald, and Scott have been selected for brief review here. Seagraves (1967; pp. 11-24) in an article on the tobacco and oil industries, suggested applicability of a risk--return model to describe motivation of investors in resources. He did not pursue the
matter in the detailed manner of Sharpe (1970) in finance theory, but did suggest that risk and return have a positive relationship.

McDonald (1967, pp. 209, 287) considered taxation with regard to theoretical supply effects caused by percentage depletion and intangibles expensing. He observed that time based depreciation and depletion will not affect production. One interesting theoretical proof provides a conclusion that percentage depletion shifts production toward the present. McDonald proceeded to consider arguments regarding whether or not the corporate income tax is shifted forward and regarding the impact of value added taxes and property taxes on mining.

Scott (1967) provided an analysis which includes graphical cost curve theory from microeconomics. Scott suggested that the output of mining should be "tilted" toward the present time. He added this tilt is preferable to a constant production rate. Apparently this suggestion is somewhat revolutionary at the time of this book, based on the remarks of Gaffney (1967, p. 3) and Scott's own remarks (1967, p. 25).

The Scott rationale rests on the premise that a constant production rate ties up capital in "illiquid form." Scott (1967, p. 45) pointed out that the optimal rate of output is a balance between time preference, with interest and other carrying costs providing the equilibrium. Scott was really producing the Hotelling conclusions from some different starting points. It is interesting to note that the Hotelling context was that the interest rate was a
rationing and conservation phenomenon, while Scott used the same concept to criticize an overly conservative production rate.

Scott showed the rising value of the ore remaining in a mine is an application of the "user cost" principle which was first named and described by Keynes (1936, Chapter 6, Appendix). In short, "user costs" describes the capital costs when capital is put to a certain specified use; i.e., the change in value of the capital stock over a measured period, less maintenance expenditures, plus net additions to the capital stock. Keynes suggested applicability to natural resources of the user costs concept in two ways. First, investment capital used by a resource exploiter was subject to the user costs. Second, the resources themselves, to the extent that they are used up today instead of tomorrow represent user costs. Scott (1953) also elaborates the user cost concept. Scott (1967, pp. 33-35) suggested that computationally, the interest factor described by Hotelling is a user cost.

R. L. Gordon (1967) suggested an analysis in which past production is allowed to influence the model results. Also, he considers cases of increasing costs and increasing demand, but then concludes that the additional variables prevent prediction or price behavior as was possible in the constant demand and constant cost Hotelling model. Gordon also considered social welfare and contends that efficient social managers will always minimize production costs, but independently owned mines may not always do so. However, Goldsmith (1974) has offered a comment on R. L. Gordon's article which refutes the conclusion that social welfare will not necessarily coincide with the private ownership, competitive solution.
Cummings (1969, p. 208) suggested use of some recent extensions in the calculus of variations to provide a more rigorous solution to a point which he said "eluded" Hotelling, R. L. Gordon, and Scott; i.e., cost increases due to deterioration (or removal) of the resource stock leads to a stretching out of the removal process. In effect, this is the problem of going deeper into the mine. The other writers cited ran into a problem that although the direction of the effect on cost was obvious, measurement was a problem because the increasing cost function was in turn a function of prior increases. Cummings' work implies a decision rule which says: continue production at time zero until the marginal value of an increment in production at time zero equals its user costs.

M. B. Zimmerman (1977) has produced a model in which a cumulative cost function is estimated, based on the geology of remaining deposits. The procedure is to first develop a long run cost function, and from it and geological data to produce a cumulative cost function. In essence, M. B. Zimmerman has elaborated on the problem of increasing costs from going deeper into the mine. M. B. Zimmerman incorporated statistical frequency distributions into his model, for purposes of cost estimation.

The foregoing literature review has focused on natural resource investment and economics theory, as it relates to the individual firm. Literature emphasizing market and social utility functions will be briefly reviewed shortly. However, there also are some books which should be noted in passing that are essentially financial management texts or reference books for the resource exploiting firm.
For example, Banks (1976) featured sections on elementary microeconomic price theory as applicable to a mineral firm. Also, the American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc. (AIME) published a comprehensive volume in 1976 entitled Economics of the Mineral Industries. The subtitle of the book describes it as "A series of Articles by Specialists." In the vein of the literature reviewed previously, there is one selection by Morse (1971) which reviews some price theory, then reviews the Hotelling model and, finally, discusses some of the implications in a social context. The remainder of the AIME volume includes such varied topics as resources in international trade, sources of financing for mineral development, and individual mineral market and production statistical studies on many different minerals, with each mineral being allocated its own separate article.

Another group of economic literature on resources considers primarily social utility. Articles emphasizing principally the social utility aspect of resources will only be briefly reviewed here because the topic is only tangential to the main focus of this study.

Koopmans (1974) and Beckman (1974) have both shown that the higher the discount rate used in a social utility calculation, the more rapid the rate of exploitation of the resource. Note that this conclusion is similar to that of Herfindahl (1967) who said that the higher discount rate, the more rapid the rate of exploitation for a firm. The point was illustrated as "Figure 2" in connection with the Herfindahl discussion earlier in the chapter.
Dasgupta and Heal (1974) have considered a model of exhaustible resources involving substitution between the exhaustible resource and a reproducible resource. Smith (1968) has provided a general economic theory of production for both replenishable and exhaustible resources. Smith (1974) has also considered technology and substitution among resources in relation to the exhaustion of some of the resources.

Externalities involved in resource depletion are the focus in writings by Lin (1976) and Nordhaus (1974).

Sweeney (1977) has discussed optimal depletion patterns for firms in competitive price markets, and then collects the firms into a market model. He then examined "biases" with which the market must contend including depletion allowances, monopolies, externalities, price regulation, and international competition.

Scott (1976) has provided a collection of articles on resource sharing revenues in a federalism taxation situation. Some of the articles are theoretical, while others have a more descriptive and pragmatic orientation to contemporary resource problems in Canada.


A problem involved in rotating many of the above cited social utility oriented articles to finance theory is that factors which are
viewed as variables in finance are described as givens or constants in social utility theory. For example, capital costs for the firm are often assumed to be either a constant factor, or a factor which is implied to be netted out since it would be encountered uniformly by all sellers in a competitive market, or was a given factor in a monopoly market. In these studies, inflation was also assumed to be a neutral factor under the premise that inflation is only a measurement complication since in equilibrium situations prices have relevance only as relative relationships among variables.

However, as was shown in Chapter 2 of this study, income taxes and inflation can be shown to cause effects on capital costs. Therefore, extension of those effects into the natural resource framework is certainly warranted.

The focus of this chapter will now be shifted from exhaustible resources to renewable natural resources such as fish and timber. The biological life cycles of these renewable resources represent investment value which will provide perpetual returns if the resource receives proper care. Therefore, complete exhaustibility of renewable resources is a viable concern only if market factors disregard the investment value of perpetual returns. For example, when forest land is cleared in order to build houses on the land, residual use of the land has greater value than the capitalized value of perpetual returns from use of the land as a forest.

Some of the economics literature on renewable resources considers social welfare and the possible divergence between social and private costs and benefits. Heady (1950), Smith (1968), Cummings
and Burt (1969), Suzuki (1976), and Sweeny (1977) have emphasized social utility aspects of optimal production from renewable resources.

Goundry (1960) discussed forest management, the rate of return for the firm, and optimal harvest rate. H. S. Gordon (1954) and Herfindahl and Kneese (1974) have considered production equilibria and investment optimization for both society and the firm, using fisheries as examples.

It is demonstratable in theory that renewable resources can have "steady-state" harvest rates such that yields each year will be constant indefinitely (Herfindahl and Kneese, 1974, pp. 154-184). In other words, if a resource population has the capability to increase in size by a certain rate each year, then a harvest at that rate should cause the population to remain constant over time.

Herfindahl and Kneese have said that a resource owner may desire to harvest at a greater or lesser rate than the "steady-state" rate in order that the "bio-mass," or population will increase or decrease. In effect, this is disinvesting if the harvest rate is greater than the steady-state rate, and is investing if the harvest rate is less than the steady-state rate. If the bio-mass is decreased a point will be reached, termed "critical point" by Herfindahl and Kneese, beyond which the remaining stock of the resource will be too small to sustain itself and it will continue to decrease. On the other hand, if the bio-mass is increased in size, a point called the "optimal steady-state" point will be reached beyond which the resource will be too populous and the population members would be competing for normal living space and food. This competition itself may
decrease the population size or may maintain a steady state at an overpopulated level.

For example, assume that a fish hatchery has a population growing at 10% annually. If the harvest rate is also 10% the population will be stable at a steady-state solution. However, if water space and food available would support a larger population, the fishery management could forego part of its 10% harvest and the capital stock would increase. When the population would reach the point where if one more fish was added, competition would cause the population to stabilize or decline, then the steady state maximum, or optimum, has been reached. At this point if the 10% harvest rate were then begun, the largest absolute number of fish would be caught each harvest.

Herfindahl and Kneese (1974) have provided a graphical analysis of steady-state solutions with capital costs also added to the model. Figure 6 represents a function where all horizontal axis "fish caught" points are steady-state solutions, and the vertical axis represents average cost of capital services per fish caught. The capital services function is assumed to be homogenous of degree one. Point A represents the maximum steady-state catch. The upper branch of the function represents mostly unstable, disregarded, solutions. For example, to the left of point A the same quantity of fish are caught at both a high cost and a low cost; thus the high cost solution is not feasible.

Herfindahl and Kneese went on to add demand curves, such as BC in Figure 7 and BDC in Figure 8. Figure 8 focuses on the fact
Figure 6. Possible combinations of renewable resource steady states and price, cost per unit of fish caught

Figure 7. Relationship of demand curve to steady-state combinations

Figure 8. Drift from steady state towards a marginal situation

that the demand curve may not be marginal to cost on the parabola at a feasible solution, such as point C. Then, the quantity, price, cost, etc. will drift toward a marginal solution, here assumed to be point D. The parabola apparatus provides a tool which will be of analytical value in Chapter 5 to demonstrate a possible effect of inflation on steady state solutions for natural resources populations.

A different situation exists when a firm exploits a resource which is not under private ownership or government management. Without the private ownership or government management, there is not necessarily any regard for the capital value represented in the resource. The investment value of the capital does not enter anyone's return on investment calculation, so there is no steady-state concern for the renewable resource. Instead, the resource stock represents only potential revenues which will enhance earnings proportionately with the speed at which the revenues are earned. Society, the real owner of the resource, has its capital investment in the resource destroyed with little, or less than adequate, return in the process. For example, the contemporary issue of whether or not certain species of whales are being exterminated is discussed by Graves (1976) and Scheffer (1976).

Since that type of issue is a social responsibility issue, instead of an investment issue, it will not be elaborated further here.

The model which was presented in Chapter 2 and various pertinent literature reviewed in Chapters 3 and 4 will now be integrated
in Chapter 5, to show the impact of the depreciation-inflation-taxes problem on natural resources.
The discussion of the literature cited Herfindahl (1967) as saying that the price could be set by the producer as part of an optimization strategy. Quantity demanded would then be a reaction to the price which had been set by the producer. Presumably, the producer would know his optimal output quantity and the demand schedule, as cited by Hotelling (1931). The demand schedule would in fact be a collection of demand responses to various prices that could be set by the producer. As in the theory of perfectly competitive markets, the natural resource producer will be assumed to set either price or quantity.

In the following it will be demonstrated that the inflation rate and the depreciation-tax factor will be crucial in determining production. This is analogous to the discussion in Chapter 2 which has shown the importance of those factors in general investment decision of the firm (Equation [2.17]).

Borrowing from Herfindahl, the price at time \( j \), would be expressed as follows:

\[
P_j = (P_0 - c) e^{r_j} + c\]
where

\[ P_j \] is price at time \( j \),
\[ P_0 \] is price at time zero,
\[ C \] is the producer cost per unit,
\[ r \] is the interest rate.

The \((P_0 - C)\) term is the royalty term which Hotelling and Herfindahl have expanded at the interest rate, and which Hotelling said was necessary as a tradeoff between production today and production tomorrow. To express the royalty as a single variable, redefine the \((P_0 - C)\) term as \( Y \).

\[ Y = P_0 - C \] \hspace{1cm} \text{[5.2]}

Then, [5.1] can be restated as follows

\[ P_j = Ye^{rj} + C \] \hspace{1cm} \text{[5.3]}

The term \( C \), which represents the producer costs per unit, will now be made explicit for periods as \( C_j \) and the equation will be restated in the period form, again to be consistent with other period notation in Chapter 2 of this study.

\[ P_j = Y(1+r)^j + C \] \hspace{1cm} \text{[5.4]}

Next, if \( C_j \) is the cost per unit and \( Q_j \) is the quantity at time \( j \), then \( Q_j C_j \) is the aggregate cost. The analysis which follows concentrates on the inflation-tax-depreciation combination effect on
output. In this sense it is a partial equilibrium approach. Other factors and costs may influence output and/or price, but such other factors are ignored in this model. $Q_j C_j$ will therefore be defined, as taxes, in the following manner:

$$[5.5] \quad Q_j C_j = t(P_j Q_j - D_j)$$

where

$t = \text{the tax rate}$

$D_j = \text{the depreciation deduction in year } j.$

Equation [5.5] represents the fact that the taxable income of the firm is equal to the total revenue less depreciation. In reality, other costs would also be deducted, but consistent with earlier work in this study, those costs are able to be passed on to the buyer, as was shown in the review of the article by Nelson (1976b). The inflation effect on those costs, since they are current costs and the producer has the ability to increase prices, cancels out and is therefore not relevant at this point.

Now rearrange [5.5] into the following:

$$[5.6] \quad C_j = \frac{t(P_j Q_j - D_j)}{Q_j}$$

Substitute [5.6] into [5.4].

$$[5.7] \quad P_j = Y(1+r)^j + \frac{t(P_j Q_j - D_j)}{Q_j}$$
In the same manner as in the presentation which was applicable to capital budgeting, inflation at \((1+f)\) per period is added to the model. Assume that the producer can react by changing all prices under his control to keep up with inflation. However, as in the capital budgeting discussion, the nominal \(D_j\) amount cannot be changed.

\[
P_j(1+f)^j = Y(1+r)^j + \frac{t[P_j(Q_j(1+f)^j - D_j)]}{Q_j}
\]

The left-hand term is now price in nominal dollars. To put it back into constant dollars, divide by \((1+f)^j\) and the following results:

\[
P_j = Y(1+r)^j + \frac{t(P_jQ_j - D_j)}{(1+f)^jQ_j}
\]

Then, equation \([5.9]\) is rearranged into:

\[
P_j = Y(1+r)^j + tP_j - \frac{tD_j}{(1+f)^jQ_j}
\]

\[
P_j = Y(1+r)^J + tP_j - tD_j(1+f)^{-j}Q_j^{-1}
\]

Then, differentiating \(P_j\) with respect to \(Q_j\):

\[
\frac{\partial P_j}{\partial Q_j} = +tD_j(1+f)^{-j}Q_j^{-2}
\]
Rearranging:

\[ 5.13 \quad \frac{\partial P_j}{\partial Q_j} = \frac{tD_j}{(1+f)^j Q_j^2} \]

Now note that the numerator is a constant while the denominator of the term on the right includes two variables, \((1+f)^j\) and \(Q_j^2\), which would move in opposite directions if they were varied while the other terms were held constant.

In a geometrical interpretation of the partial derivative of \(P\) with respect to \(Q\), the slope of that partial represents a tangent at the point of intersection of the \(P/Q\) function and the relationship of a third variable, the \(f\) or inflation. The result is a three dimensional graph, shown in Figure 9, in which successive higher amounts of inflation will be associated with smaller amounts of the \(P/Q\) or the total cost surface (with price and/or quantity set by producer to reflect cost).

Therefore, price, as set by the producer, is a function of both the quantity \(Q\), and the inflation rate, \(f\). But this might be circular reasoning, since according to the Herfindahl/Hotelling models the purpose of the producer setting his price is to, in turn, determine the quantity to produce and sell. However, for purposes of this study, regardless of whether price or quantity is set first, it is obvious that the inflation rate theoretically will enter the decision.

At any rate, the positive partial derivative confirms that price and quantity have a positive slope for a producer's supply.
Figure 9. Three-dimensional graph of decreased amounts of price (P) or quantity (Q) associated with increased inflation (f)
function. Then, when the inflation factor is varied from 0 to some very large number, the denominator becomes very large, and so the numerator divided by the denominator becomes small. Thus the slope of the $\frac{P}{Q}$ declines. Since the Hotelling model makes a price decline illogical and impossible, the quantity must be what declined, making total cost increase. A lower quantity, will make the producer set a higher price.

The relationship being described represents a shift in the cost, i.e., supply function of the firm. Note that elasticity of demand will not be relevant to the shift since the function being shifted is the supply function.

When $P$ is at an optimum (maximum) in equation [5.13], $\frac{\partial P}{\partial Q_j} = 0$. Therefore, to find the optimum solution, set the equation equal to zero and solve for $Q_j$.

\[
0 = \frac{tD_j}{(1+f)^j (Q_j^2)}
\]

\[
Q_j^2 = \frac{tD_j}{(1+f)^j}
\]

\[
Q_j = \sqrt{\frac{tD_j}{(1+f)^j}}
\]

Clearly according to this model, output quantity has been determined by the inflation factor $(1+f)$. The relationship is inverse. A large $f$ will cause the right-hand term to be smaller. Thus, the
mathematical case can be made that production is curtailed due to price inflation, all other things held equal.

The adjustment being called for in this model could be made either in the *ex ante* sense or in response to inflation being experienced. The expectations of inflation in the future could cause the quantity adjustment being called for. Also, if the firm is experiencing inflation that is higher than anticipated, the firm could react by curtailing production. Either scenario is possible in this model.

The conclusion of the model can also be reconciled to the Herfindahl (1967), Beckman (1974), and Koopmans (1974) citations in Chapter 4 in which the point is made that increases in the discount rate accelerate production, and decreases in the discount rate decrease the rate of production. Consider equation [2.17] restated here as [5.17].

\[
[5.17] \quad -C_0 + \sum_{j=0}^{n} \frac{(1-t)(R_j-E_j)}{(1+r)^j} + \sum_{j=0}^{n} \frac{tD_j}{(1+r)^j(1+f)^j} = 0
\]

Simplify the equation by writing it for a one period project, and then solve for \( r \):

\[
[5.18] \quad -C_0 + \frac{(1-t)(R-E)}{(1+r)} + \frac{tD}{(1+r)(1+f)} = 0
\]

\[
[5.19] \quad -C_0(1+r) + (1-t)(R-E) + \frac{tD}{(1+f)} = 0
\]
\[ (1-t)(R-E) + \frac{t_D}{1+f} = G_0(1+r) \]

\[ \frac{(1-t)(R-E) + \frac{t_D}{1+f}}{G_0} = -1 = r \]

Clearly, increases in $f$, the inflation rate, will decrease the real return. Therefore, as in [5.16] the same conclusion is reached, that increases in inflation will be a depressant to production, due to the accompanying decrease in the real rate of return.

In the case of renewable resources, consider again the parabola of the Herfindahl and Kneese (1974) model that was illustrated in Figures 6, 7, and 8 in Chapter 4. Given that the parabola represents capital costs per fish caught, an increase in capital costs, which has been demonstrated in Chapters 2 and 5 to occur in an inflation, will shift the parabola upward. If the demand curve had intersected the parabola at the maximum steady-state catch before inflation is added to the model, then the new intersection becomes a point below the optimal steady-state solution. If before the inflation the demand curve intersected the parabola to the left of the optimal steady-state point, then the new intersection will be even further to the left.

Then, considering the point made by Herfindahl and Kneese regarding a non-marginal intersection (Figure 8 in Chapter 4) in which there is a tendency for the solution to drift to the left to become marginal, the inflation triggered movement would be a likely motivation for such adjustment.
In short, a shift of the parabola upward will necessarily cause an intersection with demand at some quantity of production (or harvest) that is less than the production which occurred before the shift.

Therefore, it is hypothesized, based on the theoretical model which has been presented, that the production rate in natural resource exploitation will be related to inflation, and the relationship is hypothesized to be a negative relationship. The hypothesis is applicable to both exhaustible and renewable resources. That hypothesis will be tested empirically in the next chapter.
CHAPTER 6

THE CASE OF THE COPPER INDUSTRY

Empirical research on the hypothesized relationship between production and inflation does not have the luxury of being able to have a control group and an experimental group, so that the independent variables can be varied by the researcher's manipulation. There is simply no way to hold all variables constant and then vary inflation at will to test the reaction of management. Instead, research will have to be based on time series data. A model form that is appropriate for these constraints is linear multiple regression. If several of the significant independent variables can be identified and quantified and a sufficient number of observations can be obtained, the effects of the independent variable or variables can be measured. It is obvious that in regard to the problem that has been discussed in this study there are several variables that will affect production of a commodity. Thus, two variable linear regression or correlations are too naive to permit meaningful results, and multiply regression procedures will give improved results. The specifics of the regression model will be presented in a subsequent section.

In regard to the institutional organizations on which the research is based, recall that the theoretical model in Chapter 2 has implications for the individual project, the firm, and the aggregate market of an industry. Thus, if a firm reacted to increased inflation by increasing production in some or all of the capital
projects of the firm, then increased production would be empirically measurable in some projects, in some firms, and in the market as a whole. Therefore, project, firm, and market are the possible organizational levels to be considered.

The first alternative, the individual capital project (e.g., a mine, oil well) presents several problems. A specific project may have a capacity limitation or other technological constraints that would prevent management from varying production at will. An example in this regard would be the fact that in an open-pit mine the overburden has practical constraints on its rate of removal. Depending on the shape of the ore body and the mining method to be employed, often the first several years of production involve a gradual build-up to higher levels. A mature mine, on the other hand, would probably be much better on which to focus, but again technological problems at one location may give results confounded by total demand faced by the firm at all of its operating sites. In addition, a focus on individual projects would be subject to sorting out what amount of production increase that could be attributed to various factors of cost and demand.

The second alternative, the firm, may be more suitable. If the firms being studied are large enough that several mature projects are being operated simultaneously by a firm, then these firms will have fairly precise control over production volume. The potential pitfalls are again possible technological bottlenecks. It is impossible to conceive of a firm of any size in any industry that would be immune to short-run problems that could influence output. Also, as
was implied by saying "large" enough above, if a firm is comparatively small or operates only one or two projects, then this firm is more vulnerable to technological problems at those projects.

The last alternative for organizational focus is the aggregate industry for a particular resource. If random demand fluctuations can be assumed to more or less cancel each other out, if certain firms do respond to inflation with production increases, and other firms do not, the increase should still show up in the aggregate data. If a sufficiently large number of observations is used, and the regression model uses several important variables, then the aggregate industry is probably the best place to test for the hypothesized relationship.

The selection of a specific natural resource industry to provide the empirical data also has several considerations that serve to make some industries preferable to others.

The price of the resource "in the ground" (per Hotelling, 1931) versus the fixed cost of extraction should be such that the resource producers would have an incentive to increase production. Since the hypothesized relationship reflects increase in capital costs for fixed investment, then an industry which has heavy fixed investments relative to the value of the resource "in the ground" would be the most likely to exhibit the hypothesized relationship.

Renewable resources were shown in the chapter which reviewed resources literature to have less susceptibility to an increase in the exploitation rate due to the steady state life cycle which gives perpetual income properties to the renewable resource. It is
certainly possible that some exploiters of renewable resources would react, there is less theoretical logic for that strategy. Therefore, renewable resources will not be considered for a source of empirical data.

The resource from which empirical data is obtained should have a fairly broad market so that more significance can be given to the results. Also, resources with broad markets can be more easily linked to general demand measures in the economy. It seems more appealing to estimate demand for something like copper or iron ore rather than titanium or lithium.

Resources that are mostly produced as by-products should be avoided. It would be difficult to quantify the amount that is being produced in its own right. For example, the Duval Sierrita mine near Tucson, Arizona, is supposedly profitable only because both copper and molybdenum are obtained from the ore (Navin, 1978, p. 321).

There should be good domestic production data, and the bulk of the domestic consumption should be produced here. This stipulation is to prevent strong influences from Third World countries which may be using their mines as a device to stimulate the national economy. In fact, some observers conclude that these nations actually need to produce as much as they can (Wade, 1974). Also, compilation of foreign production data can be avoided if a largely domestic industry is selected.

Minerals by Component" reports that of the 34 items listed, molybdenum, copper, lead, bromine, clays, mica, phosphate, sand and gravel, stone, sulfur, talc, and salt were less than 20% imported in 1976. The list has 19 other nonfuel minerals which were over 20% imported: iron ore, chromite, cobalt, manganese, nickel, tungsten, antimony, cadmium, mercury, platinum group, tin, titanium, uranium, zinc, asbestos, barite, fluorspar, gypsum, and potash. "Aluminum" is shown on the list as 14% imported, however bauxite from which alumina is made is over 80% imported and was not on the list. The mere technicality that aluminum is processed from alumina in the United States is not relevant for the purposes of this study, so bauxite should be included as one of the imported group. Two other items on the list, pig iron and steel ingot, were 1% and 9% imported, respectively; since these are refined products, their status is not all that relevant here.

Iron ore was 37% imported, but this is due mostly to cheaper foreign supplies rather than shortage (Wade, 1974, p. 186). If fuels would have been included on the list, coal and dry natural gas would have qualified as less than 20% imports, but crude oil would not. Note the word "principal" in the title of the list. The list is not intended to be all inclusive. For example, graphite, pumice, silver, gold, and lime are not on the list.

Of the various criteria which have been listed here, copper, lead, natural gas, and coal appear to meet the criteria rather well. Natural gas is regulated at the burner tip, in interstate pipelines and in the field. The field price regulation in particular distorts price demand and supply, such that natural gas should be eliminated
as a candidate for analysis. Of the three remaining, copper was selected principally because of the availability of a demand and production model.

In construction of a testable model for the copper industry, inflation is only one of many factors that may influence production.

Demand and price factors, production costs, competition from substitute products, labor problems, technological capacity, technological production problems, environmental protection legislation, marketing strategy, scrap sales, government stockpile policy, and management's attitude toward risk are all relevant influences in copper production (Navin, 1978). Indeed, the complexity of such factors is apparent in the better known econometric models of the copper industry, such as Fisher, Cootner, and Baily (1972) and Charles River Associates (1970).

Since the development, per se, of an aggregate production function for copper is not the purpose of this study, the procedure to be followed in dealing with the various independent variables is to selectively use or duplicate an existing copper production function, and then to add inflation to the variables already in the model to test whether or not additional variability is being explained by the inflation term.

The existing model which was chosen for that purpose is the production function of the Short Run Copper Forecasting Model (SRCFM) of the Division of Economic and Business Research at The University of Arizona. A detailed description of the quarterly version of this model has been given by Taylor (1979). This model is comprised of
employment, price, production, production cost, demand, and the like for the copper industry. Altogether, there are about four hundred equations in the complete SRCFM.

The production function, in particular, uses three variables in a multiple regression equation: 1) the real price of copper; 2) the demand for copper; and 3) a labor strike variable. Other variables were tried and/or considered, but multicolinearity and other problems caused the variables in the production function to be restricted to those indicated, according to Alberta Charney,\(^1\) researcher in the Division of Economic and Business Research who did the principal work on the production function.

The following specific quantitative definitions of those variables are as follows, according to Alberta Charney and Taylor (1976, pp. 135-145) or Taylor (1975, pp. 23-33):

1. The real price of copper is the domestic refinery price quoted in the *Engineering and Mining Journal*, or *American Metals Markets*, divided by the Wholesale Price Index (now Producer Price Index).

2. The demand for copper is the United States constant dollar fixed investment plus constant dollar gross auto product, minus dollar constant dollar gross auto product arising from producer durable equipment investment.

\(^{1}\)All subsequent citations of Charney refer to a number of conferences with her.
3. The labor strike variable is foregone production, estimated by average production rate in the two quarters both before and after the strike.

4. The dependent variable, production, is defined specifically as production of refined copper (produced from domestic ores, foreign ores, and scrap) plus net imports of refined copper.

Some elaboration of those factors is appropriate. The domestic refinery price is preferred to the London Metal Exchange price because the London price is rather erratic and at times represents a rather thin market, and at other times is not really an accurate representation of what copper is really being sold for in various places in the world (Taylor, 1975, p. 15). The domestic refinery price also has advantages in that it already reflects some influences of copper scrap, aluminum, and aluminum scrap competition. On the other hand, the official price of aluminum was found to be a very poor regression variable when it was used directly in the model (Taylor, 1975, p. 14; Taylor, 1976, p. 126).

The possibility of serious market disturbances from abroad was discounted by the developers of SRCFM. They cited that from 1950 to 1972 the direction of change in copper stocks in the Free World outside the United States was the same for 16 years, and for many of the other 7 years the absolute changes were relatively small (Taylor, 1975, p. 24; Taylor, 1976, p. 136).

The strike variable stated above is the approach which was used in the quarterly SRCFM production function, according to Alberta
Charney, the strike variable defined in this way is different from the form of the "dummy strike variable" mentioned by Taylor (1979), pp. 418-420; 1975, p. 28), which was used in other parts of the SRCFM. In that other form, the "dummy strike variable" was measured by using a coefficient of one for each month that the industry was on strike. Taylor (1975, p. 28) said that this other approach to measuring the strike effects did not perform well as was anticipated, probably because there is catch-up activity after a strike, and some overproduction and stock-piling before a strike, in anticipation of it.

In summary, in this study, the intent is to use the three variables of the SRCFM production function, with some minor modifications as may be appropriate, plus inflation, in a multiple regression test for goodness of fit. The SRCFM operates in a slightly different content, that of predicting production and the other various data (employment and price, for example) using interrelated predictions. In other words, the SRCFM price of copper used in the SRCFM production function would be the predicted price being produced by another segment of the model. However, this study is not attempting to predict future data, but rather to determine if the past data could be described somewhat better with the additional factor in the model.

In order to use the SRCFM concepts in this study, the following specifics are relevant regarding data and data sources.

The data period selected was 1947-1978. The starting date is significant in that World War II influences would not be present, and data series readily available in the United States Department of Commerce publications improve greatly beginning with the 1947 date.
A sample of 32 years should provide adequate degrees of freedom to secure statistically significant observations.

Annual data were eventually selected to be used in the regressions. The SCRFM has both quarterly and annual versions, but the researchers usually use the quarterly version for their purposes (Charney, conference; Taylor, 1979). However, after some exploratory work with both the quarterly and the annual data, it was decided to first use the annual data here as there may be some quarterly disturbances in certain of the inflation variables which could complicate an unbiased interpretation of the results.

The real refinery price for this study was calculated by obtaining the nominal prices from the appropriate journals, and dividing by the Producer Price Index (formerly the Wholesale Price Index).


The labor strike variable was developed from the annual mine production in Business Statistics (1977 Edition, p. 150) and certain 1978 and 1979 issues of the Survey of Current Business. Inasmuch as the SRCFM is a quarterly model in the form described previously, and the study here is using annual data, the strike variable was developed by determining the average monthly production for operating months in each year when there was a strike, and then multiplying this number
by the number of months of the strike in order to get the foregone production. For example, the 1959 calculation is as follows. There was a 3.66 month strike, so 12.00 months less 3.66 equals 8.34 operating months. Mine production was 824,800 short tons. Then,

\[
\frac{824,800}{8.31} = 98,896 \text{ monthly production implied.}
\]

The 98,896 is then multiplied by 3.66 to get 361,960 as the strike variable for that year. This procedure was performed for all years in which strikes occurred.

The data for the dependent variable is the model, i.e., total production supply, is from *Business Statistics* (1977 Edition, pp. 150-151) and certain issues of the *Survey of Current Business*. Calculations include the foreign production and scrap, just as required in the SRCFM. Since extraction from mines is a central topic of this study, it could be asserted that including additional sources of supply (scrap, imports) as does the SRCFM, might be logically incorrect. However, it was decided to retain the SRCFM production supply definition in this study for three reasons:

1. The SRCFM researchers had concluded that these other factors did not seem to influence domestic production materially.
   (This point was cited earlier in connection with refinery price.)

2. The strike variable could possibly be interrelated with those other supply factors, and would cause multicolinearity with the strike variable.
3. Foreign producers, including foreign operations of domestic producers, would also probably react to the hypothesized production-inflation relationship, if it does exist.

Thus, leaving out the imports, etc., from the total production supply could actually distort the results rather than improve them. In fact, on an exploratory basis, United States mine production was used in place of the total production supply definition, and the results were not as significant as what will be shown in regressions in this study.

Inflation, the additional independent variable being added to the model, can be defined in numerous ways for empirical purposes. Two general types of time series which can be used for this purpose include: actual inflation measures and predictor series. The former include the various government inflation indices such as the Consumer Price Index. The latter type would include such time series as interest rates, which have been shown to be predictors of inflation.

Furthermore, the mention of a "prediction of inflation" implies a corollary issue, that of the duration of the time lag between the prediction of the inflation, and the validation of the inflationary expectations by actual changes in the price level. Lags between interest rates and price inflation, with the interest rates as the lagging factor, have been a well-researched topic in the economics literature, with writings by Fisher (1930), Yohe and Karnosky (1969) and Sargent (1973), among others. Yohe and Karnosky have shown that lags are probably only a few months for the major
part of the lag relationship. On the other hand, Fisher found lags with a duration of several years. Fama (1975, 1976, and 1977) has suggested that the 1953-1971 period Treasury bill rates have been good predictions of inflation, and that the real rate of return has been approximately constant throughout that period. However, the Fama conclusion and methodology have been questioned by Carlson (1977), Joines (1977), and Nelson and Schwert (1977) who drew different conclusions from his data.

It is also intuitive that going forward beyond the dates of Fama's sample period would make the Treasury bill rate less appealing as a predictor of anticipated inflation. During the 1970s there are many instances when the Treasury bill rate is less than the concurrently observed actual inflation rate. Should such evidence be interpreted as a prediction that investors were expecting an immediate drop to zero inflation? Most of the rational investors buying Treasury bills would readily agree that over the three month life of the bills, it would be unlikely that inflation would be halted. So in effect, for whatever reasons these investors were settling for a negative real rate of return. The Treasury bill apparently had some appeal, perhaps liquidity or low risk, which encouraged them to do so. The Federal Reserve's Regulation Q may be involved here. Low ceilings on bank savings accounts would encourage savers who wanted minimal risk to look elsewhere, and the only risk-free alternative was the Treasury bill. Consequently, rates were artificially low. Treasury bills are also readily purchased by foreign investors, which may cause yields to be unrealistic predictors of inflation.
As alternative inflation concepts for this study, the federal funds rates and 90-day prime bankers' acceptances will also be used. These two time series will not have the problem of outside influences. These two series will represent true auction markets such as exist for the Treasury bills, but the restriction of the clientele to bankers and dealers should remove the outside influences noted.

The crux of the issue here is the nature of the expectations of the rational entrepreneur. If the entrepreneur increases or decreases production in response to inflation, with all other factors held constant, then that decision must have been based on some perception of inflation. The empirical problem is to tell what and where that perception came from. In other words, the measurement problem involves a question of what news of inflation the entrepreneur might react to, and then what time span would be required for the expected change to be implemented. For instance, there may be quite a media influence present from the publication of the various government price indices such as the Consumer Price Index. Such an index could be used as a guide for anticipations on the premise that the monthly and quarterly reporting of the inflation rate are trend predictions, depending on the acceleration or deceleration of inflation apparent in the statistics.

Alternately, the influence may come from the media and other sources in response to predictor series such as interest rate changes. Perhaps the entrepreneur may be reacting directly to the factors which are influencing the interest rates.
Perhaps it could even be argued that the entrepreneur merely responds to changes in sales trends. The sales trend may result from changes in interest rates, inflation, business conditions, or other forces. These factors, however, may be one step removed. In other words, from the standpoint of the question of whether or not production would change in direct response to inflation, is the inflation just a symptom and not a cause? The empirical model which is being developed here confronts this issue by incorporating a demand variable in the model other than the inflation variable. If the inflation variable was merely a symptom a high degree of negative correlation between demand and inflation would be expected. The two factors would exhibit high multicolinearity and the predictability of the model would not be improved with the addition of the inflation factor alone. If the variable for inflation adds to the predictability of the model, it may be viewed as a causal factor.

If the entrepreneur does react and changes production, the length of the time lag between the formation of inflationary expectations and an observable decrease in production is an empirical question. If annual production data are used for the model, and the time lag is only a few days or few weeks the lag would fall within the data points. Whatever effect exists would show up as a concurrent observation. On the other hand, if the lag is on the order of several months to a year, then lagged annual data are appropriate. Ideally, inflation should be matched with the production which it encouraged or discouraged, as per the hypothesis. In the copper industry there should not be too much lag between a management
decision to step up production and getting the production increase if certain mines and concentrating-smelting operations were not operating at capacity. If concentrator-smelter expansions are necessary, however, then the lag becomes a matter of years, if the firm desired to expand production. Decreasing production would not entail this problem.

Therefore, in the specific statistics which follow, there are both concurrent and 1-year lag comparisons. In other words, the production of year $N$ is compared with the inflation statistic for time $N$ for the concurrent comparison, and production of year $N$ is compared with inflation for year $N - 1$ for the lagged comparison.

In order to investigate the effects of both the actual inflation measures (government indices, and the predictor series, i.e., interest rates), seven different series are selected for use as the inflation factors. Each series was considered on both a concurrent and a 1-year lagged comparison basis. The seven statistical series which were used to measure inflation are as follows. Abbreviations are included which will be used in subsequent tables.


5. **Federal Funds Rates (Fed Funds)**—Average annual percentage rates reported in the Federal Reserve Bulletin.

6. **Prime Bankers Acceptances (Prime)**—Average annual percentage rates reported in Federal Reserve Bulletin, for 90 day acceptances.

7. **Treasury Bill Rates (T-bill)**—Average annual percentage rates for new issues of three month Treasury bills, as reported in Business Statistics (1979, p. 83).

Inasmuch as all the multiple regression variables have now been discussed, the following linear multiple regression model is
now suggested to summarize the linear relationship between the inflation rate and copper production:

\[ QP_j = a + b_1 DC_j + b_2 CP_j + b_3 SV_j + b_4 IF_j + U \]

where

- \( QP_j \) is the total production supply in period \( j \), expressed in short tons, as per SRCFM definition.
- \( a \) is a constant in the regression equation.
- \( b_1 \) to \( b_4 \) are the regression coefficients of the independent variables.
- \( DC_j \) is the demand for copper, as defined by the SRCFM, for period \( j \).
- \( CP_j \) is the real domestic refinery price of copper, as defined by the SRCFM for period \( j \).
- \( SV_j \) is the strike variable, measured in foregone production, in period \( j \), computed as has been discussed.
- \( IF_j \) is the inflation variable. The seven data series suggested are to be used alternately for this term, on both concurrent and lagged bases.
- \( U \) is the disturbance term.

The disturbance term \( U \) is assumed to be random with a normal distribution. The sign and significance of the coefficients will relate directly to the hypothesis and SRCFM. The independent variables which have been taken from the SRCFM will have the sign, coefficient and significance level which led to their being included.
in that model. Of special note here is the strike variable. Since its coefficient has been written in the equation as "$+b_3$" the strike variable data must be entered as negative numbers, since production and strikes would be inversely related.

If the new independent variable, inflation, has a positive sign, the interpretation is that inflation encouraged production. If inflation has a negative sign, then the hypothesis described in mathematical terms in Chapter 5 is confirmed, and it can be concluded that inflation discouraged production. Given that the results are statistically significant, a rough estimate can also be made of the impact on short run production.

Sixteen multiple regressions were run using the Statistical Package for the Social Sciences (SPSS). The seven statistical series were run on both a concurrent and a lagged basis for annual data. Also, for comparison, regressions were run on the copper data without any inflation measure in the equation. Table 6 shows the regression results for the eight concurrent regressions. Table 7 shows the results for the regressions in which production was lagged a year behind the inflation measure. In the tables which follow, short descriptive names will be used for the various regression variables: Production for $QP_j$, Demand for $DC_j$, Price for $CP_j$, Strike for $SV_j$ and Inflation for the $IF_j$.

In Table 6, among the concurrent regressions, the $t$ statistics for the Consumer Price Index and the Gross National Product Deflator are significant at the 5% level. The Machinery and Equipment Index is significant at the 1% level. The Producer Price Index is
Table 6. Summary statistics for regression models of copper production and concurrent inflation measures: 1947-1978 annual data

\[ \text{QP}_j + a + b_1 \text{DC}_j + b_2 \text{CP}_j + b_3 \text{SV}_j + b_4 \text{IF}_j + U \]

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Constant ($t$ stat.)</th>
<th>Demand $b_1$ ($t$ stat.)</th>
<th>Price $b_2$ ($t$ stat.)</th>
<th>Strike $b_3$ ($t$ stat.)</th>
<th>Inflation $b_4$ ($t$ stat.)</th>
<th>$R^2$</th>
<th>$R^2$ Change</th>
<th>$R^2$</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>704.193 (6.312)</td>
<td>7.690 (6.961)**</td>
<td>6.424 (1.724)</td>
<td>.267 (2.650)*</td>
<td>-2.767 (-2.064)*</td>
<td>.914</td>
<td>.014</td>
<td>.900</td>
<td>2.022</td>
</tr>
<tr>
<td>Equip. Ind.</td>
<td>793.947 (7.578)</td>
<td>8.176 (8.368)**</td>
<td>5.126 (1.486)</td>
<td>.266 (2.861)**</td>
<td>-3.437 (-3.024)**</td>
<td>.925</td>
<td>.038</td>
<td>.914</td>
<td>2.249</td>
</tr>
<tr>
<td>FPI(WPI)</td>
<td>732.428 (6.217)</td>
<td>6.921 (7.367)**</td>
<td>6.503 (1.679)</td>
<td>.264 (2.562)*</td>
<td>-1.824 (-1.708)†</td>
<td>.909</td>
<td>.010</td>
<td>.896</td>
<td>1.941</td>
</tr>
<tr>
<td>Fed. Funds</td>
<td>707.372 (5.339)</td>
<td>4.890 (5.767)**</td>
<td>8.098 (2.110)*</td>
<td>.228 (2.174)*</td>
<td>17.790 (1.010)</td>
<td>.903</td>
<td>.003</td>
<td>.889</td>
<td>1.712</td>
</tr>
<tr>
<td>Prime</td>
<td>588.587 (5.328)</td>
<td>6.085 (7.245)**</td>
<td>10.590 (2.802)*</td>
<td>.233 (2.209)*</td>
<td>-15.343 (-1.791)</td>
<td>.902</td>
<td>.002</td>
<td>.887</td>
<td>1.815</td>
</tr>
<tr>
<td>T-bill</td>
<td>613.641 (4.541)</td>
<td>5.493 (6.631)**</td>
<td>9.530 (2.477)*</td>
<td>.228 (2.134)*</td>
<td>4.850 (.199)</td>
<td>.900</td>
<td>.000</td>
<td>.884</td>
<td>1.763</td>
</tr>
<tr>
<td>None</td>
<td>621.549 (6.116)</td>
<td>5.566 (10.702)**</td>
<td>9.597 (2.711)*</td>
<td>.232 (2.214)*</td>
<td>N/A</td>
<td>.899</td>
<td>N/A</td>
<td>.889</td>
<td>1.721</td>
</tr>
</tbody>
</table>

** Significance at the 1% level
* Significance at the 5% level
† Significance at the 10% Level

D-W Refers to the Durbin-Watson statistic
N/A Indicates that an item is not applicable
$R^2$ Is the $R^2$ adjusted for the degrees of freedom
$R^2$ Change is the increase in $R^2$ due to the inclusion of the inflation measure in the regression
Table 7. Summary statistics for regression models of copper production and inflation measure of prior years: annual data for 1947 inflation and 1948 production through 1977 inflation and 1978 production

\[ QP_j = a + b_1 DC_j + b_2 CP_j + b_3 SV_j + b_4 IF_j + U \]

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Constant (a) (t stat.)</th>
<th>Demand (b_1) (t stat.)</th>
<th>Price (b_2) (t stat.)</th>
<th>Strike (b_3) (t stat.)</th>
<th>Inflation (b_4) (t stat.)</th>
<th>(R^2)</th>
<th>(R^2) Change</th>
<th>(\bar{R}^2)</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>748.311 (6.458)</td>
<td>8.024 (7.203)**</td>
<td>5.802 (1.574)</td>
<td>.272 (2.752)*</td>
<td>-3.594 (-2.366)*</td>
<td>.917</td>
<td>.028</td>
<td>.904</td>
<td>2.100</td>
</tr>
<tr>
<td>Equip. Ind.</td>
<td>757.112 (7.304)</td>
<td>8.607 (8.290)**</td>
<td>4.564 (1.309)</td>
<td>.263 (2.878)**</td>
<td>-4.269 (-3.167)**</td>
<td>.927</td>
<td>.043</td>
<td>.916</td>
<td>2.471</td>
</tr>
<tr>
<td>GNP Defl.</td>
<td>727.307 (6.671)</td>
<td>8.323 (7.085)**</td>
<td>5.719 (1.574)</td>
<td>.271 (2.774)**</td>
<td>-4.915 (-2.493)*</td>
<td>.919</td>
<td>.030</td>
<td>.906</td>
<td>2.150</td>
</tr>
<tr>
<td>Prime</td>
<td>537.279 (5.697)</td>
<td>6.925 (10.949)**</td>
<td>11.987 (3.734)**</td>
<td>.235 (2.539)*</td>
<td>-61.338 (-2.962)**</td>
<td>.925</td>
<td>.024</td>
<td>.913</td>
<td>1.709</td>
</tr>
<tr>
<td>T-bill</td>
<td>510.103 (4.485)</td>
<td>6.503 (9.048)**</td>
<td>11.435 (3.204)**</td>
<td>.221 (2.172)*</td>
<td>-29.473 (-1.667)</td>
<td>.909</td>
<td>.010</td>
<td>.895</td>
<td>1.697</td>
</tr>
<tr>
<td>None</td>
<td>597.098 (5.711)</td>
<td>5.655 (10.716)**</td>
<td>9.794 (2.762)**</td>
<td>.227 (2.172)*</td>
<td>N/A</td>
<td>.899</td>
<td>N/A</td>
<td>.888</td>
<td>1.766</td>
</tr>
</tbody>
</table>

** Significance at the 1% level
* Significance at the 5% level
D-W Refers to the Durbin-Watson statistic
N/A Indicates an item that is not applicable
\(\bar{R}^2\) Is the \(R^2\) adjusted for the degrees of freedom
\(R^2\) Change is the increase in \(R^2\) due to the inclusion of the inflation measure in the regression
significant at the 10% level. Conspicuous by their lack of significance in the concurrent regressions are the three interest rate inflation predictor series.

The inclusion of the price indices increased the coefficient of multiple determination by 1% to 2%. The descriptive power of the regression equation was apparently improved by the price indices used as an inflation proxy.

The regression coefficients are negative for the price indices which had significant *t* values. This is consistent with the hypothesis that the relationship between inflation and production is negative.

The Durbin-Watson bounds for 5% are 1.16 and 1.74. None of the price indices regressions have any autocorrelation, as they are all outside those bounds. However, it should be noted that the data when run without an inflation measure in the model resulted in 1.721 Durbin-Watson which may represent some autocorrelation. Then, when the price indices were added, the autocorrelation disappeared.

When lagged regressions were run, as shown in Table 7, the Machinery and Equipment Index and prime bankers' acceptances are significant at the 1% level. The other price indices are significant at the 5% level. The federal funds rates and Treasury bill rates just miss being significant at the 10% level.

The lagged regressions increased the coefficient of multiple determination in amounts ranging from about 2% to 4% for the price indices and the prime bankers' acceptances. The $R^2$ increases for federal funds and Treasury bills were about one percent. These effects are higher than the $R^2$ changes obtained with the concurrent
regressions. All of the inflation measures in the lagged regressions had negative coefficients as hypothesized.

The Durbin-Watson statistics of the four price indices were over 2.0 and, therefore, no autocorrelation exists in those regressions. (The 5% bounds are 1.16 and 1.74.) The three regressions of interest rate series and the regression of the data with inflation omitted all have Durbin-Watson statistics of somewhat below 1.74. Therefore, some autocorrelation exists for those series.

The relevant correlation coefficients for the variables are given in Tables 8 and 9. The only serious possibility for multicollinearity appears to be between demand and the various inflation measures being introduced into the model. In spite of possible multicollinearity, regression models are improved if the mean square error is reduced by the addition of an additional variable. Since the $R^2$ has been increased when the inflation variables were included, reduction of mean square error is present.

Also tolerance statistics will detect multicollinearity. A tolerance of .000 indicates the presence of another factor already in the regression which completely explains the variance of the entering variable, and a 1.000 tolerance indicates that there is such variable present. Tolerance statistics are summarized in Table 10, and the values are acceptable for time series economic data.

The variables which were borrowed from SRCFM produced results which were generally what was expected with regard to significance and coefficient. The only problem with those variables is that real refinery price lost significance in several regressions, for
Table 8. Correlation coefficients for regression variables, 1947-1978, using annual data for concurrent production and inflation

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Variable</th>
<th>Production</th>
<th>Demand</th>
<th>Price</th>
<th>Strike</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>Demand</td>
<td>.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.739</td>
<td>.877</td>
<td>.355</td>
<td>.177</td>
</tr>
<tr>
<td>Equip. Ind.</td>
<td>Demand</td>
<td>.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.716</td>
<td>.874</td>
<td>.376</td>
<td>.073</td>
</tr>
<tr>
<td>PPI(WPI)</td>
<td>Demand</td>
<td>.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.665</td>
<td>.804</td>
<td>.277</td>
<td>.013</td>
</tr>
<tr>
<td>GPN Defl.</td>
<td>Demand</td>
<td>.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.768</td>
<td>.901</td>
<td>.412</td>
<td>.062</td>
</tr>
<tr>
<td>Fed. Funds</td>
<td>Demand</td>
<td>.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.869</td>
<td>.876</td>
<td>.689</td>
<td>.144</td>
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<tr>
<td></td>
<td>Price</td>
<td>.693</td>
<td>.620</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Strike</td>
<td>.036</td>
<td>.165</td>
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Table 9. Correlation coefficients for regression variables, 1947-1978, using annual data; production lagged one year after inflation

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<th>Variable</th>
<th>Correlation Matrix</th>
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<td>Strike</td>
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<td>Inflation</td>
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<td>Production</td>
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</tr>
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<td></td>
<td>Demand</td>
<td>.599</td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.148</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.158</td>
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<tr>
<td></td>
<td>Strike</td>
<td>.035</td>
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<td>Equip. Ind.</td>
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<td>Strike</td>
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<td></td>
<td>Inflation</td>
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<td>Production</td>
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<td>Demand</td>
<td>.599</td>
</tr>
<tr>
<td></td>
<td>Price</td>
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</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.158</td>
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<td></td>
<td>Strike</td>
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<td>.158</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.008</td>
</tr>
<tr>
<td>GNP Defl.</td>
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<td>Price</td>
<td>.681</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
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<td></td>
<td>Inflation</td>
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<td>Production</td>
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<td>Demand</td>
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</tr>
<tr>
<td></td>
<td>Price</td>
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<tr>
<td></td>
<td>Strike</td>
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<td></td>
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<td>Strike</td>
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<td>Inflation</td>
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<tr>
<td></td>
<td>Production</td>
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<td></td>
<td>Demand</td>
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<td>Price</td>
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</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.158</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.149</td>
</tr>
<tr>
<td>Prime</td>
<td>Demand</td>
<td>.926</td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.681</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.020</td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.661</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Demand</td>
<td>.599</td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.148</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.158</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.149</td>
</tr>
<tr>
<td>T-bill</td>
<td>Demand</td>
<td>.926</td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>.681</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.020</td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.721</td>
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<tr>
<td></td>
<td>Production</td>
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<td></td>
<td>Demand</td>
<td>.599</td>
</tr>
<tr>
<td></td>
<td>Price</td>
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</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.158</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.148</td>
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</table>
Table 10. Tolerance statistics for inflation measures used in copper model regressions, 1947-1978 annual data

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Concurrent Regressions</th>
<th>Lagged Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>.178</td>
<td>.222</td>
</tr>
<tr>
<td>Equip. Ind.</td>
<td>.236</td>
<td>.225</td>
</tr>
<tr>
<td>PPI(WPI)</td>
<td>.264</td>
<td>.386</td>
</tr>
<tr>
<td>GNP Defl.</td>
<td>.188</td>
<td>.189</td>
</tr>
<tr>
<td>Fed. Funds</td>
<td>.197</td>
<td>.369</td>
</tr>
<tr>
<td>Prime</td>
<td>.215</td>
<td>.355</td>
</tr>
<tr>
<td>T-bill</td>
<td>.170</td>
<td>.311</td>
</tr>
</tbody>
</table>
example the four price index regressions in the concurrent models in Table 6.

The lag factor remained as a variable which invited additional testing. Therefore, the data were also tested on a quarterly basis for the period from 1955 through 1978. Complete quarterly data were readily available beginning in 1955 rather than 1947. The Treasury bill rates, the federal funds rates, and the prime bankers' acceptance rates were all tested in the same manner as with the annual data. Six different lag assumptions were tested for the lag between inflation and the level of production. The six lag periods were: concurrent (no lag), one quarter, two quarters, three quarters, four quarters, and five quarters.

Neither the Treasury bill rates nor the federal funds rates provided any statistically significant results for any of the six lag assumptions. The closest that any of them came to being significant was the four quarter lag assumption for both Treasury bills and federal funds rates which were both significant at about the 25% error level.

However, the prime bankers' acceptance rates were significant at the 5% level for a regression of the fourth previous quarter's interest rate with the production in a particular quarter. Table 11 summarizes the regressions of the prime bankers' acceptance rates. Short descriptive names will be used as before with the annual data regressions. For the prime bankers' acceptance rates the short descriptive name will be "Prime" followed by the number of quarters.
Table 11. Summary statistics for regression models of inflation and lagged copper production using quarterly data, 1955-1978

\[ QP_j = a + b_1 DC_j + b_2 CP_j + b_3 SV_j + b_4 IF_j + U \]

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Constant (t stat.)</th>
<th>Demand (t stat.)</th>
<th>Price (t stat.)</th>
<th>Strike (t stat.)</th>
<th>Inflation (t stat.)</th>
<th>( R^2 )</th>
<th>( R^2 ) Change</th>
<th>( R^2 )</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime One</td>
<td>333.412 (6.435)</td>
<td>1.663 (-2.905)</td>
<td>-2.905</td>
<td>.479 (7.490)**</td>
<td>-3.980 (-.867)</td>
<td>.688</td>
<td>.003</td>
<td>.675</td>
<td>1.805</td>
</tr>
<tr>
<td>Prime Two</td>
<td>341.022 (6.662)**</td>
<td>1.550 (-3.087)</td>
<td>-3.087**</td>
<td>.481 (7.489)**</td>
<td>-1.213 (-.285)</td>
<td>.687</td>
<td>.000</td>
<td>.673</td>
<td>1.809</td>
</tr>
<tr>
<td>Prime Three</td>
<td>341.867 (6.711)</td>
<td>1.570 (-3.089)</td>
<td>-3.089**</td>
<td>.482 (7.497)**</td>
<td>-1.626 (-.403)</td>
<td>.687</td>
<td>.001</td>
<td>.673</td>
<td>1.816</td>
</tr>
<tr>
<td>Prime Four</td>
<td>344.279 (6.855)**</td>
<td>1.705 (-3.017)</td>
<td>-3.017**</td>
<td>.478 (7.550)**</td>
<td>-6.384 (-1.657)$$</td>
<td>.696</td>
<td>.009</td>
<td>.682</td>
<td>1.843</td>
</tr>
<tr>
<td>Prime Five</td>
<td>351.414 (7.045)</td>
<td>1.749 (-3.135)</td>
<td>-3.135**</td>
<td>.476 (7.600)**</td>
<td>-8.174 (-2.161)*</td>
<td>.702</td>
<td>.015</td>
<td>.689</td>
<td>1.868</td>
</tr>
<tr>
<td>None</td>
<td>352.377 (6.953)</td>
<td>1.689 (-3.223)</td>
<td>-3.223**</td>
<td>.486 (7.635)**</td>
<td>-6.056 (-1.571)</td>
<td>.695</td>
<td>.008</td>
<td>.681</td>
<td>1.862</td>
</tr>
<tr>
<td></td>
<td>352.458 (6.756)</td>
<td>1.50 (-3.142)</td>
<td>-3.142**</td>
<td>.481 (7.521)**</td>
<td>N/A</td>
<td>.686</td>
<td>N/A</td>
<td>.676</td>
<td>1.806</td>
</tr>
</tbody>
</table>

* Significance at the 1% level  
** Significance at the 5% level  
$ Significance at the 10% level  
D-W Refers to the Durbin-Watson test  
\( R^2 \) Is the \( R^2 \) adjusted for the degrees of freedom  
N/A Indicates that an item is not applicable  
\( R^2 \) Change in the increase of \( R^2 \) due to inclusion of the inflation measure in the model
of lag in the particular regression. For comparison, Table 11 also contains a regression which omits the inflation variable.

The "Prime Three" regression resulted in the inflation factor being significant at the 10% level. The "Prime Four" regression is significant almost at the 10% level. Therefore, these results are consistent with the results found in the annual data. The one-year lag found in that data would approximate the four quarters lag which was found in the quarterly test.

The $R^2$ increase is 1.5% for the four quarters lag model. There is sufficient mean square error reduction to justify including the inflation variable. Mean square error reduction is from 3,161.5 to 3,040.3. The descriptive power of the model is improved by including the inflation variable.

The Durbin-Watson statistics are all 1.8 or greater for the regressions in Table 11. The 5% bounds are 1.58 and 1.75, so it may be concluded that no autocorrelation exists in this quarterly test.

The correlation coefficients for the quarterly prime bankers' acceptance rates and production data are shown in Table 12. The extent of multicolinearity is substantially less in this quarterly data than in the annual data. The correlation coefficients for demand and prime bankers' acceptance rates are approximately .62 for the significant regressions, while coefficients between .80 and .90 were typical for demand and the inflation factors in Tables 8 and 9.

The troublesome factor in this quarterly data is the negative coefficient for real refinery price. The other variables all have
Table 12. Correlation coefficients of regression variables: 1955-1978 quarterly data with comparative assumptions of lag between inflation and copper production

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Variable</th>
<th>Production</th>
<th>Demand</th>
<th>Price</th>
<th>Strike</th>
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<td>Price</td>
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<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
<td>-.015</td>
<td>.063</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.439</td>
<td>.738</td>
<td>-.025</td>
<td>-.015</td>
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<td>Prime One</td>
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<td>.685</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>-.329</td>
<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
<td>-.015</td>
<td>.063</td>
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<td></td>
<td>Inflation</td>
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<td>-.047</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>-.329</td>
<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
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<td>.063</td>
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<td></td>
<td>Inflation</td>
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<tr>
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<td>Price</td>
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<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
<td>-.015</td>
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<td></td>
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<tr>
<td></td>
<td>Price</td>
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<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
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<td>Inflation</td>
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<td>.616</td>
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<td>-.033</td>
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<tr>
<td>Prime Five</td>
<td>Demand</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>-.329</td>
<td>-.262</td>
<td></td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>Strike</td>
<td>.418</td>
<td>-.015</td>
<td>.063</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inflation</td>
<td>.376</td>
<td>.624</td>
<td>-.201</td>
<td>-.018</td>
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</tbody>
</table>
the same sign as in the annual data. A positive coefficient for real refinery price in the annual data and a negative coefficient in the quarterly data create an ambiguous status for that variable. The worst ambiguity is, of course, for the predictive uses of the copper model. Intuitively, a higher real price should call forth more production, and therefore, the positive coefficient in the annual data is as expected. The negative coefficient in the quarterly data is not consistent with the theory. A detailed digression investigating why this may have occurred is beyond the scope of this study.

In order to confirm the credibility of the quarterly data, the four quarters lag regression was re-run with Price left out. For purposes of this study, if the same result for the inflation factor is obtained both with and without real refinery price in the model, then the same conclusions can be drawn as with the annual data. The results of this regression are in Table 13. Prime Four continued to be significant at the 5% level, although the significance did drop slightly, as might be expected with one less variable. Comparable t statistics from Tables 11 and 13 are 2.161 and 2.073, respectively.

However, removing the Price variable caused the Durbin-Watson statistic to drop to 1.663 which is within the 1.58 to 1.75 bounds. Therefore, real refinery price did add predictability to the model, even though it has the negative coefficient. Real price has in fact declined over the 1955-1978 period, and production has risen; thus, the result which was obtained is the same one which would have resulted from a simple correlation of real price and production.
Table 13. Summary statistics for regression model of inflation and copper production lagged four quarters, real refinery price omitted, 1955-1978

\[ QP_j = a + b_1DC_j + b_2CP_j + b_3SV_j + b_4IF_j + U \]

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Demand</th>
<th>Price</th>
<th>Strike</th>
<th>Inflation</th>
<th>( R^2 )</th>
<th>( R^2 ) Change</th>
<th>( R^2 ) D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime Four</td>
<td>213.408</td>
<td>1.866</td>
<td>N/A</td>
<td>.464</td>
<td>-8.214</td>
<td>.669</td>
<td>.015</td>
<td>.658</td>
</tr>
<tr>
<td>(Price</td>
<td>(8.495)</td>
<td>(10.364)**</td>
<td>N/A</td>
<td>(7.084)**</td>
<td>(2.073)*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Omitted</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Significance at the 1% level  
** Significance at the 5% level  
D-W Refers to the Durbin-Watson test  
\( R^2 \) is \( R^2 \) adjusted for degrees of freedom  
N/A Indicates that the item is not applicable  
\( R^2 \) Change in the increase of \( R^2 \) due to inclusion of the inflation measure in the model
Finally, a possibility of a relation among the various quarters was considered. In other words, a test was made to see if including two or more quarters of inflation prediction data simultaneously would improve the regression. For example, pairings such as one quarter lag and four quarters lag inflation data were both regressed in the copper production model simultaneously. However, preliminary results indicated excessive multicollinearity among the inflation prediction statistics. This approach was tried for Treasury bills, federal funds, and prime bankers' acceptance rates.
CHAPTER 7

SUMMARY

The results of the regression of annual data indicate that the interest rate inflation predictor series worked well on a lagged basis. The various price indices were significant on both a concurrent and a lagged basis, but the statistics were slightly better for the lagged. It was logical that predictions of inflation and the inflation which was predicted would correlate. Therefore, it is consistent that the interest rate series were significant on a lagged basis and the price indices were significant on a concurrent basis in the annual data.

However, the prior period's price index regressing negatively with a high degree of significance was not anticipated. With an explicit demand variable in the model, this result should not be merely a proxy for demand. Perhaps the media effect of the announcement of various price index increases or decreases did have a real effect.

The results found here are consistent with the hypothesis that there is a negative interaction between production and inflation. The amount of that interaction, as estimated by regression, is thousands of tons in the case of the copper industry. For example, the annual lagged regression model has a coefficient of -41.338 on Prime in Table 7. Thus, a 1% change in that rate theoretically means
a difference of 41,338 tons of production.\footnote{Production data were entered as thousands of tons and Prime was entered as interest points and tenths of a point. For example, 10.3\% would be 10.3 in the data. Then, \(-41,338 \times 1.0 \times 1,000 = 41,338\) given above.} A 5\% inflation rate would imply a negative 200,000 tons.

In light of the various commodity shortages realized in the United States economy during the 1970s, the inflation of the 1970s may have exacerbated the shortages, if the empirical results of the copper industry can be generalized to other commodities. Also, it could be argued that with inflation causing real prices to decline, there would be shortages at the lower real prices as markets adjust.

While the thirty-one year sample selected for copper in the lagged annual regressions did show significant effects, it remains for additional empirical testing to confirm this effect for other commodities. However, as was explained herein, significant empirical problems exist in applying this methodology to other commodities.

The point of departure in this study which caused the theoretical mathematics to show that there is a negative effect on production, was based on the historical cost-based depreciation required in federal income taxation. More rapid writeoffs may lessen the impact of this problem. Yet, rapid writeoffs will not always be a solution. To elucidate the point, assume that a firm is allowed a 100\% writeoff at the time of investment in a project, but suppose that there is not sufficient income from the project as yet, and the firm shows a tax loss because of the large writeoff. Then the depreciation deduction becomes a carryback or carryforward, still denominated in nominal
dollars. If it is a carryforward, still denominated in nominal dollars, real present value will decline until the writeoff is completely used. This problem cannot be completely prevented in all cases unless a methodology such as indexation is used. Countries which have used indexation have had administrative problems. Yet it would seem that if the indexation is confined to the depreciation deduction, the calculation would be no more complex than the various accelerated depreciation methods now in use. Price-level adjustments with index numbers are a common topic in accounting texts.

The interrelationship of investment and production effects of inflation are also significant in a macroeconomic sense. The inverse relation between inflation and investment was shown in the literature to have several capital budgeting effects. For example, replacements would be deferred and the capital/labor ratio would be shifted toward the labor intensive technologies. This study has also focused on the apparent negative relation between inflation and production. Those various effects have interrelated macroeconomic effects because the factors involved are all a part of aggregate demand and aggregate production in the economy.

Finally, it is certainly possible that the inverse relation between production and inflation which was shown for the copper industry may also be due to other factors besides the depreciation-inflation-tax effect. That possibility has been left for future research.
APPENDIX A

REAL RATES OF RETURN

The equation attributed to Fisher (1930) in Chapter 1 can be derived as follows, where $P$ is principal, $h$ is the nominal rate of interest, $r$ is the real rate of interest, and $f$ is the inflation rate.

\[
\frac{P(1+h)}{(1+f)} = P(1+r)
\]

\[
\frac{(1+h)}{(1+f)} = (1+r)
\]

\[
\frac{(1+h)}{(1+f)} + 1 = r
\]

One significant fact which was made explicit in Fisher's work is the real rate of return is less than the nominal rate minus the inflation rate.

Consider the second equation above. The following results are obtained by solving for the nominal rate:

\[
(1+h) = (1+r)(1+f)
\]

\[
l+r+f+rf
\]

\[
h = r + f + rf
\]

Clearly, the nominal rate represents the sum of the real rate, the inflation rate, and the cross-products of the real and inflation rates.
Returns on securities can be expressed in an equivalent expression. Without considering inflation, let $D$ be dividends on stock (or bond interest), $V_t$ be market value at time $t$, and $r$ the rate of return. Then,

$$\frac{D + V_{t+1}}{V_t} = r$$

in which case $r$ would be both nominal and real returns. If inflation represented by $f$ is added to the model, and $r$ represents real return,

$$\frac{D + V_{t+1}}{(1+f)} = r$$

which is equivalent to

$$\frac{D + V_{t+1}}{(1+f)} = r$$

Since the $(D+V_{t+1})/V_t$ represents nominal return plus 1, then $(1+f)$ from the first equation could be substituted and the two equations would be identical.
APPENDIX B

ILLUSTRATION OF INFLATION-TAX EFFECT
USING ACCELERATED DEPRECIATION

Liberalized depreciation may overcome part of the impact of the inflation-tax effect which would have otherwise occurred, but it is impossible to completely overcome the problem without one hundred percent expensing and/or indexation. If expensing was used, indexation would still have to be allowed for carryovers when the expense cannot be used in one year.

For example, consider equipment which is "3 year property" under the Accelerated Cost Recovery System (ACRS) in the Economic Recovery Tax Act of 1981. The appropriate writeoff percentages are 25%, 38% and 37% for assets acquired in 1981-1984.

Therefore, for an asset costing $10,000 the following numbers would occur in working out the components of

\[ \sum_{j=0}^{n} \frac{tD_j}{(1+f)^j(1+r)^j} \]

which was developed in Chapter 2.

<table>
<thead>
<tr>
<th>Year</th>
<th>ACRS Percentages</th>
<th>Depreciation for Tax Return</th>
<th>Taxes Saved @ 46% rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25%</td>
<td>2500</td>
<td>1150</td>
</tr>
<tr>
<td>2</td>
<td>38%</td>
<td>3800</td>
<td>1748</td>
</tr>
<tr>
<td>3</td>
<td>37%</td>
<td>3700</td>
<td>1703</td>
</tr>
</tbody>
</table>
If the desired real rate of return is 10% and there is no inflation, the tax saved should be discounted 10%. If there is a 10% real rate of return and a 10% inflation rate, then \((1.1)(1.1) = 1.21\) and 21% should be used for discounting.

<table>
<thead>
<tr>
<th>Year</th>
<th>Tax Saved (above)</th>
<th>10% Discount Factors</th>
<th>Present Values Using 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1150</td>
<td>.9091</td>
<td>1,045</td>
</tr>
<tr>
<td>2</td>
<td>1748</td>
<td>.8264</td>
<td>1,445</td>
</tr>
<tr>
<td>3</td>
<td>1702</td>
<td>.7513</td>
<td>1,279</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sum of Present Values</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3,769</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Tax Saved (above)</th>
<th>21% Discount Factors</th>
<th>Present Values Using 21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1150</td>
<td>.8264</td>
<td>950</td>
</tr>
<tr>
<td>2</td>
<td>1748</td>
<td>.6830</td>
<td>1,194</td>
</tr>
<tr>
<td>3</td>
<td>1702</td>
<td>.5645</td>
<td>961</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sum of Present Values</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3,105</td>
</tr>
</tbody>
</table>

Obviously, any inflation rate will cause some present value less than that which would be obtained if only the real rate was used for discounting.

If indexation were permitted the depreciation would be nominally adjusted for the inflation rate.
The 21% discount factors would then be used to put the nominal tax saved into real present value terms.

<table>
<thead>
<tr>
<th>Year</th>
<th>Original Depreciation</th>
<th>Indexation at 10% Inflation</th>
<th>Indexed Depreciation</th>
<th>Nominal Tax Saved @ 46%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,500</td>
<td>(1.10)</td>
<td>2,750</td>
<td>1,265</td>
</tr>
<tr>
<td>2</td>
<td>3,800</td>
<td>(1.10)^2</td>
<td>4,598</td>
<td>2,115</td>
</tr>
<tr>
<td>3</td>
<td>3,700</td>
<td>(1.10)^3</td>
<td>4,925</td>
<td>2,266</td>
</tr>
</tbody>
</table>

Note that the indexation cancelled the effect of the inflation, and the present value sum is the same as that which was originally obtained for the case of no inflation.
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